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We value your feedback and recommendations.

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GRADE 7 MATH TEACHING GUIDE

Lesson I: SETS: AN INTRODUCTION
Pre-requisite Concepts: Whole numbers
Objectives:
In this lesson, you are expected to:
1. Describe and illustrate
   a. well-defined sets;
   b. subsets;
   c. universal set, and;
   d. the null set.
2. Use Venn Diagrams to represent sets and subsets.

NOTE TO THE TEACHER:
This lesson looks easy and fast to teach but don’t be deceived. The introductory concepts are always crucial. What differentiates a set from any group is that a set is well defined. Emphasize this to the students.

You may vary the activity by giving them a different set of objects to group. You may make this into a class activity by showing a poster of objects in front of the class or even make it into a game. The idea is for them to create their own well-defined groups according to what they see as common characteristics of elements in a group.

Lesson Proper:
A.
   I. Activity

Below are some objects. Group them as you see fit and label each group.
Answer the following questions:

a. How many groups are there?
b. Does each object belong to a group?
c. Is there an object that belongs to more than one group? Which one?

**NOTE TO THE TEACHER:**
You need to follow up on the opening activity hence, the problem below is important. Ultimately, you want students to apply the concepts of sets to the set of real numbers.

The groups are called sets for as long as the objects in the group share a characteristic and are thus, well defined.

**Problem:** Consider the set consisting of whole numbers from 1 to 200. Let this be set U. Form smaller sets consisting of elements of U that share a different characteristic. For example, let E be the set of all even numbers from 1 to 200.

Can you form three more such sets? How many elements are there in each of these sets? Do any of these sets have any elements in common?

Did you think of a set with no element?

**NOTE TO THE TEACHER:**
Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

**Important Terms to Remember**
The following are terms that you must remember from this point on.
1. A set is a well-defined group of objects, called elements that share a common characteristic. For example, 3 of the objects above belong to the set of head covering or simply hats (ladies hat, baseball cap, hard hat).
2. The set $F$ is a subset of set $A$ if all elements of $F$ are also elements of $A$. For example, the even numbers 2, 4, and 12 all belong to the set of whole numbers. Therefore, the even numbers 2, 4, and 12 form a subset of the set of whole numbers. $F$ is a proper subset of $A$ if $F$ does not contain all elements of $A$.
3. The universal set $U$ is the set that contains all objects under consideration.
4. The null set $\emptyset$ is an empty set. The null set is a subset of any set.
5. The cardinality of a set $A$ is the number of elements contained in $A$.

**Notations and Symbols**
In this section, you will learn some of the notations and symbols pertaining to sets.
1. Uppercase letters will be used to name sets and lowercase letters will be used to refer to any element of a set. For example, let $H$ be the set of all objects on page 1 that cover or protect the head. We write

$$H = \{\text{ladies hat, baseball cap, hard hat}\}$$
This is the listing or roster method of naming the elements of a set.

Another way of writing the elements of a set is with the use of a descriptor. This is the rule method. For example, \( H = \{x \mid x \text{ covers and protects the head} \} \). This is read as “the set \( H \) contains the element \( x \) such that \( x \) covers and protects the head.”

2. The symbol \( \emptyset \) or \( \{ \} \) will be used to refer to an empty set.
3. If \( F \) is a subset of \( A \), then we write \( F \subseteq A \). We also say that \( A \) contains the set \( F \) and write it as \( A \supseteq F \). If \( F \) is a proper subset of \( A \), then we write \( F \subset A \).
4. The cardinality of a set \( A \) is written as \( n(A) \).

II. Questions to Ponder (Post-Activity Discussion)

\[ \text{NOTE TO THE TEACHER:} \]
\[ \text{It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.} \]

Let us answer the questions posed in the opening activity.
1. How many sets are there? 
   \[ \text{There is the set of head covers (hats), the set of trees, the set of even numbers, and the set of polyhedra. But, there is also a set of round objects and a set of pointy objects. There are 6 well-defined sets.} \]

2. Does each object belong to a set? Yes.

3. Is there an object that belongs to more than one set? Which ones? 
   \[ \text{All the hats belong to the set of round objects. The pine trees and two of the polyhedra belong to the set of pointy objects.} \]

III. Exercises

Do the following exercises. Write your answers on the spaces provided:
1. Give 3 examples of well-defined sets.

   Possible answers: The set of all factors of 24, The set of all first year students in this school, The set of all girls in this class.

2. Name two subsets of the set of whole numbers using both the listing or roster method and the rule method.

Example:
Listing or Roster Method:
\[ E = \{0, 2, 4, 6, 8, \ldots\} \]
\[ O = \{1, 3, 5, 7, \ldots\} \]

Rule Method:
\[ E = \{2x \mid x \text{ is a whole number}\} \]
\[ O = \{2x+1 \mid x \text{ is a whole number}\} \]
3. Let \( B = \{1, 3, 5, 7, 9\} \). List all the possible subsets of \( B \).

\[
\{ \}, \{1\}, \{3\}, \{5\}, \{7\}, \{9\}, \{1, 3\}, \{1, 5\}, \{1, 7\}, \{1, 9\}, \{3, 5\}, \{3, 7\}, \{3, 9\}, \{5, 7\}, \{5, 9\}, \{7, 9\}, \{1, 3, 5\}, \{1, 3, 7\}, \{1, 3, 9\}, \{3, 5, 7\}, \{3, 5, 9\}, \{5, 7, 9\}, \{1, 5, 7\}, \{1, 5, 9\}, \{1, 7, 9\}, \{3, 7, 9\}, \{1, 3, 5, 7\}, \{1, 3, 5, 9\}, \{1, 5, 7, 9\}, \{3, 5, 7, 9\}, \{1, 3, 7, 9\}, \{1, 3, 5, 7, 9\}\]

\(- 32\) subsets in all.

4. Answer this question: How many subsets does a set of \( n \) elements have? \( \text{There are} 2^n \text{ subsets in all.} \)

B. Venn Diagrams

**NOTE TO THE TEACHER:**

A lesson on sets will not be complete without using Venn Diagrams. Note that in this lesson, you are merely introducing the use of these diagrams to show sets and subsets. The extensive use of the Venn Diagrams will be introduced in the next lesson, which is on set operations. The key is for students to be able to verbalize what they see depicted in the Venn Diagrams.

Sets and subsets may be represented using Venn Diagrams. These are diagrams that make use of geometric shapes to show relationships between sets.

Consider the Venn diagram below. Let the universal set \( U \) be all the elements in sets \( A, B, C \) and \( D \).

![Venn Diagram](image)

Each shape represents a set. Note that although there are no elements shown inside each shape, we can surmise how the sets are related to each other. Notice that set \( B \) is inside set \( A \). This indicates that all elements in \( B \) are contained in \( A \). The same with set \( C \). Set \( D \), however, is separate from \( A, B, C \). What does it mean?

**Exercise**

Draw a Venn diagram to show the relationships between the following pairs or groups of sets:
1.  \( E = \{2, 4, 8, 16, 32\} \)
    \( F = \{2, 32\} \)

   **Sample Answer**

   ![Venn Diagram](image)

2.  \( V \) is the set of all odd numbers
    \( W = \{5, 15, 25, 35, 45, 55, \ldots\} \)

   **Sample Answer**

   ![Venn Diagram](image)

3.  \( R = \{x \mid x \text{ is a factor of 24}\} \)
    \( S = \{\} \)
    \( T = \{7, 9, 11\} \)

   **Sample Answer:**

   ![Venn Diagram](image)

**NOTE TO THE TEACHER:**

End the lesson with a good summary.

**Summary**

In this lesson, you learned about sets, subsets, the universal set, the null set and the cardinality of the set. You also learned to use the Venn diagram to show relationships between sets.
Lesson 2.1: Union and Intersection of Sets  
Time: 1.5 hours

Pre-requisite Concepts: Whole Numbers, definition of sets, Venn diagrams

Objectives:
In this lesson, you are expected to:
1. Describe and define
   a. union of sets;
   b. intersection of sets.
2. Perform the set operations
   a. union of sets;
   b. intersection of sets.
3. Use Venn diagrams to represent the union and intersection of sets.

Note to the Teacher:
Below are the opening activities for students. Emphasize that just like with the whole number, operations are also used on sets. You may combine two sets or form subsets. Emphasize to students that in counting the elements of a union of two sets, elements that are common to both sets are counted only one.

Lesson Proper:

I. Activities

Answer the following questions:
1. Which of the following shows the union of set A and set B? How many elements are in the union of A and B?
2. Which of the following shows the intersection of set A and set B? How many elements are there in the intersection of A and B?

Here's another activity:
Let

\[ V = \{ 2x | x \in I, 1 \leq x \leq 4 \} \]
\[ W = \{ x^2 | x \in I, -2 \leq x \leq 2 \} \]

What elements may be found in the intersection of V and W? How many are there? What elements may be found in the union of V and W? How many are there?

Do you remember how to use Venn Diagrams? Based on the diagram below, (1) determine the elements that belong to both A and B; (2) determine the elements that belong to A or B or both. How many are there in each set?
NOTE TO THE TEACHER:
Below are important terms, notations and symbols that students must remember. From here on, be consistent in your notations as well so as not to confuse your students. Give plenty of examples and non-examples.

Important Terms/Symbols to Remember

The following are terms that you must remember from this point on.

1. Let A and B be sets. The **union** of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

   An element $x$ belongs to the union of the sets A and B if and only if $x$ belongs to A or $x$ belongs to B. This tells us that
   $$A \cup B = \{x \mid x \text{ is in } A \text{ or } x \text{ is in } B\}$$

   Venn diagram:

   ![Venn Diagram](image)

   **Note to the Teacher:**
   Explain to the students that in general, the **inclusive OR** is used in mathematics. Thus, when we say, “elements belonging to A or B”, that includes the possibility that the elements belong to both. In some instances, “belonging to both” is explicitly stated when referring to the intersection of two sets. Advise students that from here onwards, OR is used inclusively.

2. Let A and B be sets. The **intersection** of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

   An element $x$ belongs to the intersection of the sets A and B if and only if $x$ belongs to A and $x$ belongs to B. This tells us that
A \cap B = \{ x \mid x \text{ is in } A \text{ and } x \text{ is in } B \}

Venn diagram:

Sets whose intersection is an empty set are called disjoint sets.

3. The cardinality of the union of two sets is given by the following equation:

\[ n(\ A \cup B\ ) = n(\ A\ ) + n(\ B\ ) - n(\ A \cap B\ ). \]

II. Questions to Ponder (Post-Activity Discussion)

NOTE TO THE TEACHER
It is important for you to go over the answers of your students posed in the opening activities in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered. Below are the correct answers to the questions posed in the activities.

Let us answer the questions posed in the opening activity.

1. Which of the following shows the union of set A and set B? Set 2. This is because it contains all the elements that belong to A or B or both. There are 8 elements.

2. Which of the following shows the intersection of set A and set B? Set 3. This is because it contains all elements that are in both A and B. There are 3 elements.

In the second activity:

\[ V = \{ 2, 4, 6, 8 \} \]
\[ W = \{ 0, 1, 4 \} \]

Therefore, \( V \cap W = \{ 4 \} \) has 1 element and \( V \cup W = \{ 0, 1, 2, 4, 6, 8 \} \) has 6 elements. Note that the element \{ 4 \} is counted only once.

On the Venn Diagram: (1) The set that contains elements that belong to both A and B consists of two elements \{1, 12\}; (2) The set that contains elements that belong to A or B or both consists of 6 elements \{1, 10, 12, 20, 25, 36\}.

NOTE TO THE TEACHER:
Always ask for the cardinality of the sets if it is possible to obtain such number, if only to emphasize that

\[ n(\ A \cup B\ ) \neq n(\ A\ ) + n(\ B\ ). \]
because of the possible intersection of the two sets. In the exercises below, use every opportunity to emphasize this. Discuss the answers and make sure students understand the “why” of each answer.

III. Exercises

1. Given sets A and B,

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students who play the guitar</td>
<td>Students who play the piano</td>
</tr>
<tr>
<td>Ethan Molina</td>
<td>Mayumi Torres</td>
</tr>
<tr>
<td>Chris Clemente</td>
<td>Janis Reyes</td>
</tr>
<tr>
<td>Angela Dominguez</td>
<td>Chris Clemente</td>
</tr>
<tr>
<td>Mayumi Torres</td>
<td>Ethan Molina</td>
</tr>
<tr>
<td>Joanna Cruz</td>
<td>Nathan Santos</td>
</tr>
</tbody>
</table>

determine which of the following shows (a) union of sets A and B; and (b) intersection of sets A and B?

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethan Molina, Chris Clemente, Angela, Dominguez, Mayumi Torres, Joanna Cruz</td>
<td>Mayumi Torres, Ethan Molina, Chris Clemente</td>
<td>Mayumi Torres, Janis Reyes, Chris Clemente, Ethan Molina, Nathan Santos</td>
<td>Ethan Molina, Chris Clemente, Angela, Dominguez, Mayumi Torres, Joanna Cruz, Janis Reyes, Nathan Santos</td>
</tr>
</tbody>
</table>

Answers: (a) Set 4. There are 7 elements in this set. (b) Set 2. There are 3 elements in this set.

2. Do the following exercises. Write your answers on the spaces provided:

Given the sets above, determine the elements and cardinality of:

a. \( A \cup B = \{0, 1, 2, 3, 4, 6, 8\} \); \( n(A \cup B) = 7 \)
b. \( A \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\} \); \( n(A \cup C) = 8 \)
c. \( A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \); \( n(A \cup B \cup C) = 10 \)
d. \( A \cap B = \{0, 2, 4\} \); \( n(A \cap B) = 3 \)
e. \( B \cap C = \emptyset \); \( n(B \cap C) = 0 \)
f. \( A \cap B \cap C = \emptyset \); \( n(A \cap B \cap C) = 0 \)
g. \((A \cap B) \cup C = \{0, 1, 2, 3, 4, 5, 7, 9\} \); \( n((A \cap B) \cup C) = 8 \)
NOTE TO THE TEACHER:

In Exercise 2, you may introduce the formula for finding the cardinality of the union of 3 sets. But, it is also instructive to give students the chance to discover this on their own. The formula for finding the cardinality of the union of 3 sets is:

\[
\text{n}(A \cup B \cup C) = \text{n}(A) + \text{n}(B) + \text{n}(C) - \text{n}(A \cap B) - \text{n}(A \cap C) - \text{n}(B \cap C) + \text{n}(A \cap B \cap C).
\]

3. Let \( W = \{x \mid 0 < x < 3\} \), \( Y = \{x \mid x > 2\} \), and \( Z = \{x \mid 0 \leq x \leq 4\} \).

Determine (a) \((W \cup Y) \cap Z\); (b) \(W \cap Y \cap Z\).

Answers:

Since at this point students are more familiar with whole numbers and fractions greater than or equal to 0, use a partial real numberline to show the elements of these sets.

(a) \((W \cup Y) \cap Z = \{x \mid 0 < x \leq 4\}\)

(b) \(W \cap Y \cap Z = \{x \mid 2 < x < 3\}\)

NOTE TO THE TEACHER:

End with a good summary. Provide more exercises on finding the union and intersection of sets of numbers.

Summary

In this lesson, you learned about the definition of union and intersection of sets. You learned also how to use Venn diagrams to represent the unions and the intersection of sets.
Lesson 2.2: Complement of a Set

Pre-requisite Concepts: sets, universal set, empty set, union and intersection of sets, cardinality of sets, Venn diagrams

About the Lesson:
The complement of a set is an important concept. There will be times when one needs to consider the elements not found in a particular set A. You must know that this is when you need the complement of a set.

Objectives:
In this lesson, you are expected to:
1. Describe and define the complement of a set;
2. Find the complement of a given set;
3. Use Venn diagrams to represent the complement of a set.

NOTE TO THE TEACHER
Review the concept of universal set before introducing this lesson. Emphasize to the students that there are situations when it is more helpful to consider the elements found in the universal set that are not part of set A.

Lesson Proper:
I. Problem
In a population of 8000 students, 2100 are Freshmen, 2000 are Sophomores, 2050 are Juniors and the remaining 1850 are either in their fourth or fifth year in university. A student is selected from the 8000 students and it is not a Sophomore, how many possible choices are there?

Discussion
Definition: The complement of a set A, written as \( A' \), is the set of all elements found in the universal set, U, that are not found in set A. The cardinality \( n(A') \) is given by
\[
    n(A') = n(U) - n(A)
\]

Examples:
1. Let \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \), and \( A = \{0, 2, 4, 6, 8\} \).
Then the elements of $A'$ are the elements from $U$ that are not found in $A$.
Therefore, $A' = \{1, 3, 5, 7, 9\}$

2. Let $U = \{1, 2, 3, 4, 5\}$, $A = \{2, 4\}$ and $B = \{1, 5\}$. Then,
   
   $A' = \{1, 3, 5\}$
   
   $B' = \{2, 3, 4\}$
   
   $A' \cup B' = \{1, 2, 3, 4, 5\} = U$

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 7, 8\}$. Then,
   
   $A' = \{5, 6, 7, 8\}$
   
   $B' = \{1, 2, 5\}$
   
   $A' \cap B' = \{5, 6\}$

4. Let $U = \{1, 3, 5, 7, 9\}$, $A = \{5, 7, 9\}$ and $B = \{1, 5, 7, 9\}$. Then,
   
   $A \cap B = \{5, 7, 9\}$
   
   $(A \cap B)' = \{1, 3\}$

5. Let $U$ be the set of whole numbers. If $A = \{x \mid x$ is a whole number and $x > 10\}$, then
   
   $A' = \{x \mid x$ is a whole number and $0 \leq x \leq 10\}$.

The opening problem asks for how many possible choices there are for a student that was selected and known to be a non-Sophomore. Let $U$ be the set of all students and $n(U) = 8000$. Let $A$ be the set of all Sophomores then $n(A) = 2000$. The set $A'$ consists of all students in $U$ that are not Sophomores and $n(A') = n(U) - n(A) = 6000$. Therefore, there are 6000 possible choices for that selected student.

**NOTE TO THE TEACHER:**

Pay attention to how students identify the elements of the complement of a set. Teach them that a way to check is to take the union of a set and its complement. The union is the universal set $U$. That is, $A \cup A' = U$. Recall to them as well that $n(A \cup A') = n(A) + n(A') - n(A \cap A') = n(A) + n(A') = n(U)$ since $A \cap A' = \emptyset$ and therefore, $n(A \cap A') = 0$.

In the activity below, use Venn diagrams to show how the different sets relate to each other so that it is easier to identify unions and intersections of sets or complements or unions and intersections of sets. Watch as well the language that you use. In particular, $(A \cup B)'$ is read as “the complement of the union of $A$ and $B$”
II. Activity

Shown in the table are names of students of a high school class by sets according to the definition of each set.

<table>
<thead>
<tr>
<th>A Like Singing</th>
<th>B Like Dancing</th>
<th>C Like Acting</th>
<th>D Don’t Like Any</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasper</td>
<td>Charmaine</td>
<td>Jacky</td>
<td>Billy</td>
</tr>
<tr>
<td>Faith</td>
<td>Leby</td>
<td>Jasper</td>
<td>Ethan</td>
</tr>
<tr>
<td>Jacky</td>
<td>Joel</td>
<td>Ben</td>
<td>Camille</td>
</tr>
<tr>
<td>Miguel</td>
<td>Jezryl</td>
<td>Joel</td>
<td>Tina</td>
</tr>
<tr>
<td>Joel</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After the survey has been completed, find the following sets.

a. \( U = \)
b. \( A \cup B' = \)
c. \( A' \cup C = \)
d. \( (B \cup D)' = \)
e. \( A' \cap B = \)
f. \( A' \cap D' = \)
g. \( (B \cap C)' = \)

The easier way to find the elements of the indicated sets is to use a Venn diagram showing the relationships of \( U \), sets \( A \), \( B \), \( C \), and \( D \). Set \( D \) does not share any members with \( A \), \( B \), and \( C \). However, these three sets share some members. The Venn diagram below is the correct picture:
Now, it is easier to identify the elements of the required sets.

a. \( U = \{\text{Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Joel, Leby, Miguel, Tina}\} \)
b. \( A \cup B' = \{\text{Faith, Miguel, Joel, Jacky, Jasper, Ben, Billy, Ethan, Camille, Tina}\} \)
c. \( A' \cup C = \{\text{Jasper, Jacky, Joel, Ben, Leby, Charmaine, Jezryl, Billy, Ethan, Camille, Tina}\} \)
d. \( (B \cup D)' = \{\text{Faith, Miguel, Jacky, Jasper, Ben}\} \)
e. \( A' \cap B = \{\text{Leby, Charmaine, Jezryl}\} \)
f. \( A' \cap D' = \{\text{Leby, Charmaine, Jezryl, Ben}\} \)
g. \( (B \cap C)' = \{\text{Ben, Billy, Camille, Charmaine, Ethan, Faith, Jacky, Jasper, Jezryl, Leby, Miguel, Tina}\} \)

NOTE TO THE TEACHER
Below are the answers to the exercises. Encourage discussions among students. Take note of the language they use. It is important that students say the words or phrases correctly. Whenever appropriate, use Venn diagrams.

III. Exercises
1. True or False. If your answer is false, give the correct answer.
   Let \( U = \) the set of the months of the year
   \( X = \{\text{March, May, June, July, October}\} \)
   \( Y = \{\text{January, June, July}\} \)
   \( Z = \{\text{September, October, November, December}\} \)
a. $Z' = \{\text{January, February, March, April, May, June, July, August}\}$  **True**
b. $X' \cap Y' = \{\text{June, July}\}$  **False.** $X' \cap Y' = \{\text{February, April, August, September, November, December}\}$
c. $X' \cup Z' = \{\text{January, February, March, April, May, June, July, August, September, November, December}\}$  **True**
d. $(Y \cup Z)' = \{\text{February, March, April, May}\}$  **False.** $(Y \cup Z)' = \{\text{February, March, April, May, August}\}$.

**NOTE TO THE TEACHER**

The next exercise is a great opportunity for you to develop students’ reasoning skills. If the complement of $A$, the complement of $B$ and the complement of $C$ all contain the element $a$ then $a$ is outside all three sets but within $U$. If $B'$ and $C'$ both contain $b$ but $A'$ does not, then $A$ contains $b$. This kind of reasoning must be clear to students.

2. Place the elements in their respective sets in the diagram below based on the following elements assigned to each set:
U = \{a, b, c, d, e, f, g, h, i, j\}
A' = \{a, c, d, e, g, j\}
B' = \{a, b, d, e, h, i\}
C' = \{a, b, c, f, h, i, j\}

NOTE TO THE TEACHER:
In Exercise 3, there are many possible answers. Ask students to show all their work. This is a good opportunity for them to argue and justify their answers. Engage them in meaningful discussions. Encourage them to explain their work. Help them decide which diagrams are correct.

3. Draw a Venn diagram to show the relationships between sets U, X, Y, and Z, given the following information.

- U, the universal set contains set X, set Y, and set Z.
- \(X \cup Y \cup Z = U\)
- Z is the complement of X.
- \(Y'\) includes some elements of X and the set Z
Summary

In this lesson, you learned about the complement of a given set. You learned how to describe and define the complement of a set, and how it relates to the universal set, U and the given set.
Lesson 3: Problems Involving Sets

Time: 1 hour

Prerequisite Concepts: Operations on Sets and Venn Diagrams

Objectives:

In this lesson, you are expected to:

1. Solve word problems involving sets with the use of Venn diagrams
2. Apply set operations to solve a variety of word problems.

NOTE TO THE TEACHER

This is an important lesson. Do not skip it. This lesson reinforces what students learned about sets, set operations and the Venn diagram in solving problems.

Lesson Proper:

I. Activity

Try solving the following problem:

In a class of 40 students, 17 have ridden an airplane, 28 have ridden a boat. 10 have ridden a train, 12 have ridden both an airplane and a boat. 3 have ridden a train only and 4 have ridden an airplane only. Some students in the class have not ridden any of the three modes of transportation and an equal number have taken all three.

a. How many students have used all three modes of transportation?
b. How many students have taken only the boat?

NOTE TO THE TEACHER

Allow students to write their own solutions. Allow them to discuss and argue. In the end, you have to know how to steer them to the correct solution.

II. Questions/Points to Ponder (Post-Activity Discussion)

Venn diagrams can be used to solve word problems involving union and intersection of sets. Here are some worked out examples:

1. A group of 25 high school students were asked whether they use either Facebook or Twitter or both. Fifteen of these students use Facebook and twelve use Twitter.

   a. How many use Facebook only?
   b. How many use Twitter only?
   c. How many use both social networking sites?

   Solution:

   Let $S_1 =$ set of students who use Facebook only
   $S_2 =$ set of students who use both social networking sites
   $S_3 =$ set of students who use Twitter only
The Venn diagram is shown below

Finding the elements in each region:

\[ n(S_1) + n(S_2) + n(S_3) = 25 \]
\[ n(S_1) + n(S_2) = 15 \]
\[ \text{But} \quad n(S_2) + n(S_3) = 12 \]
\[ \therefore n(S_3) = 10 \]
\[ n(S_1) = 13 \]

The number of elements in each region is shown below

2. A group of 50 students went in a tour in Palawan province. Out of the 50 students, 24 joined the trip to Coron; 18 went to Tubbataha Reef; 20 visited El Nido; 12 made a trip to Coron and Tubbataha Reef; 15 saw Tubbataha Reef and El Nido; 11 made a trip to Coron and El Nido and 10 saw the three tourist spots.
   a. How many of the students went to Coron only?
   b. How many of the students went to Tubbataha Reef only?
   c. How many joined the El Nido trip only?
   d. How many did not go to any of the tourist spots?

Solution:
To solve this problem, let
\[ P_1 = \text{students who saw the three tourist spots} \]
\[ P_2 = \text{those who visited Coron only} \]
\[ P_3 = \text{those who saw Tubbataha Reef only} \]
\( P_4 \) = those who joined the El Nido trip only
\( P_5 \) = those who visited Coron and Tubbataha Reef only
\( P_6 \) = those who joined the Tubbataha Reef and El Nido trip only
\( P_7 \) = those who saw Coron and El Nido only
\( P_8 \) = those who did not see any of the three tourist spots

Draw the Venn diagram as shown below and identify the region where the students went.

Determine the elements in each region starting from \( P_1 \).
\( P_1 \) consists of students who went to all 3 tourist spots. Thus, \( n(P_1) = 10 \).
\( P_1 \cup P_5 \) consists of students who visited Coron and Tubbataha Reef but this set includes those who also went to El Nido. Therefore, \( n(P_5) = 12 - 10 = 2 \) students visited Coron and Tubbataha Reef only.
\( P_1 \cup P_6 \) consists of students who went to El Nido and Tubbataha Reef but this set includes those who also went to Coron. Therefore, \( n(P_6) = 15 - 10 = 5 \) students visited El Nido and Tubbataha Reef only.
\( P_1 \cup P_7 \) consists of students who went to Coron and El Nido but this set includes those who also went to Tubbataha Reef. Therefore, \( n(P_7) = 11 - 10 = 1 \) student visited Coron and El Nido only.

From here, it follows that
\( n(P_2) = 24 - n(P_1) - n(P_6) - n(P_7) = 24 - 10 - 2 - 1 = 11 \) students visited Coron only.
\( n(P_3) = 18 - n(P_1) - n(P_5) - n(P_6) = 18 - 10 - 2 - 5 = 1 \) student visited Tubbataha Reef only.
\( n(P_4) = 20 - n(P_1) - n(P_5) - n(P_7) = 20 - 10 - 5 - 1 = 4 \) students visited Coron and El Nido only.

Therefore
\( n(P_8) = 50 - n(P_1) - n(P_2) - n(P_3) - n(P_4) - n(P_5) - n(P_6) - n(P_7) = 16 \) students did not visit any of the three spots.

The number of elements is shown below.
Now, what about the opening problem? Solution to the Opening Problem (Activity):
Can you explain the numbers?

Now, what about the opening problem? Solution to the Opening Problem (Activity):
Can you explain the numbers?

III. Exercises
Do the following exercises. Represent the sets and draw a Venn diagram when needed.

1. If \( A \) is a set, give two subsets of \( A \). Answer: \( \emptyset \) and \( A \)

2. (a) If \( A \) and \( B \) are finite sets and \( A \subseteq B \), what can you say about the cardinalities of the two sets?
   (b) If the cardinality of \( A \) is less than the cardinality of \( B \), does it follow that \( A \subseteq B \)?
   Answer: (a) \( n(A) < n(B) \); (b) No. Example: \( A = \{1, 2\}, B = \{2, 4, 6\} \)

3. If \( A \) and \( B \) have the same cardinality, does it follow that \( A = B \)? Explain.
   Answer: Not necessarily. Example, \( A = \{1, 2, 3\} \) and \( B = \{4, 8, 9\} \).

4. If \( A \subseteq B \) and \( B \subseteq C \). Does it follow that \( A \subseteq C \)? Illustrate your reasoning using a Venn diagram. Answer: Yes.

NOTE TO THE TEACHER
Discuss the solution thoroughly and clarify all questions your students might have. Emphasize the notation for the cardinality of a set.
5. Among the 70 kids in Barangay Magana, 53 like eating in Jollibee while 42 like eating in McDonalds. How many like eating both in Jollibee and in McDonalds? In Jollibee only? In McDonalds only?

Solution:
Let \( n(M_1) \) = kids who like Jollibee only
\( n(M_2) \) = kids who like both Jollibee and McDonalds
\( n(M_3) \) = kids who like McDonalds only

Draw the Venn diagram

Find the elements in each region

\[
\begin{align*}
 n(M_1) + n(M_2) + n(M_3) &= 70 \\
 n(M_1) + n(M_2) &= 53 \\
 n(M_2) + n(M_3) &= 42
\end{align*}
\]

\[
\begin{align*}
 n(M_3) &= 17 \\
 n(M_1) &= 28 \\
 n(M_2) &= 25
\end{align*}
\]

Check using Venn diagram
6. The following diagram shows how all the First Year students of Maningning High School go to school.

![Venn Diagram](image)

a. How many students ride in a car, jeep and the MRT going to their school? 15
b. How many students ride in both a car and a jeep? 34
c. How many students ride in both a car and the MRT? 35
d. How many students ride in both a jeep and the MRT? 32
e. How many students go to school in a car only 55 a jeep only 76 in the MRT only 67 walking 100
f. How many students First Year students of Maningning High School are there? 269

7. The blood-typing system is based on the presence of proteins called antigens in the blood. A person with antigen A has blood type A. A person with antigen B has blood type B, and a person with both antigens A and B has blood type AB. If no antigen is present, the blood type is O. Draw a Venn diagram representing the ABO System of blood typing.

A protein that coats the red blood cells of some persons was discovered in 1940. A person with the protein is classified as Rh positive (Rh+), and a person whose blood cells lack the protein is Rh negative (Rh–). Draw a
Venn diagram illustrating all the blood types in the ABO System with the corresponding Rh classifications.

NOTE TO THE TEACHER
The second problem is quite complex. Adding the 3rd set Rh captures the system without altering the original diagram in the first problem.

Summary
In this lesson, you were able to apply what you have learned about sets, the use of a Venn diagram and set operations in solving word problems.
Lesson 4.1: Fundamental Operations on Integers: Addition of Integers  
Time: 1 hour  

Pre-requisite Concepts: Whole numbers, Exponents, Concept of Integers  

Objectives:  
In this lesson, you are expected to:  
1. Add integers using different approaches;  
2. Solve word problems involving addition of integers.  

NOTE TO THE TEACHER  
This lesson is a review and deepening of the concept of addition of integers. Keep in mind that the definitions for the operations on integers must retain the properties of the same operations on whole numbers or fractions. In this sense, the operations are merely extended to cover a bigger set of numbers. We present here two models for addition that have been used to represent addition of whole numbers.  

Lesson Proper:  
I. Activity  
Study the following examples:  
   A. Addition Using Number Line  

1. Use the number line to find the sum of 6 & 5.  

   ![Number Line for 6 + 5](image)  

   On the number line, start with point 6 and count 5 units to the right. At what point on the number line does it stop?  
   It stops at point 11; hence, $6 + 5 = 11$.  

2. Find the sum of 7 and (-3).  

   ![Number Line for 7 + (-3)](image)  

   On the number line, start from 7 and count 3 units going to the left since the sign of 3 is negative.  
   At which point does it stop?  
   It stops at point 4; hence, $(-3) + (7) = 4$.  

After the 2 examples, can you now try the next two problems?  
   a. $(-5) + (-4)$  
   b. $(-8) + (5)$
NOTE TO THE TEACHER

More examples may be given if needed to emphasize an interpretation of the negative sign as a direction to the left of the number line.

We now have the following generalization:
Adding a positive integer \( n \) to \( m \) means moving along the real line a distance of \( n \) units to the right from \( m \). Adding a negative integer \( -n \) to \( m \) means moving along the real line a distance of \( n \) units to the left from \( m \).

NOTE TO THE TEACHER

Other objects might be used in this next activity. Signed tiles could be algebra tiles or counters with different colors on each side. Bottle caps are easily obtained and will be very good visual and hands-on materials.

B. Addition Using Signed Tiles

This is another device that can be used to represent integers. The tile \( + \) represents integer 1, the tile \( - \) represents -1, and the flexible \( + \) \( - \) represents 0.

Recall that a number and its negative cancel each other under the operation of addition. This means
\[
4 + (-4) = 0 \\
15 + (-15) = 0 \\
(-29) + 29 = 0
\]
In general, \( n + (-n) = (-n) + n = 0 \).

NOTE TO THE TEACHER

Get the students to model the above equations using signed tiles or colored counters.

Examples:
1. \( 4 + 5 \) \( \rightarrow \) \( + + + + \) \( \rightarrow \) four (+1) \( + \) \( + + + + + + \) \( \rightarrow \) five (+1) 
   hence, \( 4 + 5 = 9 \)

2. \( 5 + (-3) \) \( \rightarrow \) \( + + + + + \) \( \rightarrow \) \( + + + + + \) \( \rightarrow \) \( - - - \) 
   \( \rightarrow \) \( 0 \) \( \rightarrow \) \( 0 \) \( \rightarrow \) \( 0 \) 
   hence, \( 5 + (-3) = 2 \)
hence, \( 5 + (-3) = 2 + 3 + (-3) = 2 + 0 = 2 \)

3. \((-7) + (-6)\)

\[
\begin{array}{c}
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---}
\end{array}
+ \begin{array}{c}
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---}
\end{array}
\]

\[
\text{hence } (-7) + (-6) = -13
\]

Now, try these:
1. \((-5) + (-11)\)
2. \((6) + (-9)\)

Solution:
1. \((-5) + (-11)\)

\[
\begin{array}{c}
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---}
\end{array}
+ \begin{array}{c}
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---}
\end{array}
\]

\[
\text{hence, } (-5) + (-11) = -16.
\]

2. \((6) + (-9)\)

\[
\begin{array}{c}
\text{+++} \\
\text{+++}
\end{array}
\begin{array}{c}
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---} \\
\text{---}
\end{array}
\]

\[
\quad
\text{hence, } (6) + (-9) = -3.
\]

If colored counters (disks) or bottle caps are used, one side of the counter denotes “positive” while the other side denotes “negative”. For example, with counters having black and red sides, black denotes “positive”, while red denotes “negative”. For this module, we will use white instead of red to denote negative.

Examples:
1. The configurations below represent \( 5 + (-7) \)

Keeping in mind that a black disk and a white disk cancel each other, take out pairs consisting of a black and a white disk until there are no more pairs left.
This tells us that $5 + (-7) = -2$

2. Give a colored-counter representation of $(-3) + 6$

Therefore, $(-3) + 6 = 3$

The signed tiles model gives us a very useful procedure for adding large integers having different signs.

Examples:
1. $-63 + 25$
   - Since 63 is bigger than 25, break up 63 into 25 and 38.
   - Hence $-63 + 25 = -38 + (-25) + 25 = -38 + 0 = -38$

2. $724 + (-302) = 422 + 302 + (-302) = 422 + 0 = 422$

II. Questions/ Points to Ponder

Using the above model, we summarize the procedure for adding integers as follows:

1. If the integers have the same sign, just add the positive equivalents of the integers and attach the common sign to the result.

   a. $27 + 30 = + (\frac{27}{}/ + \frac{30}{})$
      = $+ (\frac{57}{})$
      = $+ 57$

   b. $(-20) + (-15) = - (\frac{20}{}/ + \frac{15}{})$
      = $- (20 + 15 )$
      = $- (35 )$
      = $- 35$
2. If the integers have different signs, get the difference of the positive equivalents of the integers and attach the sign of the larger number to the result.

a. \((38) + (-20)\)
Get the difference between 38 and 20: 18
Since 38 is greater than 20, the sign of the sum is positive.
Hence \(38 + (-20) = 18\)

b. \((-42) + 16\)
Get the difference between 42 and 16: 26
Since 42 is greater than 16, the sum will have a negative sign.
Hence \((-42) + 16 = -26\)

**NOTE TO THE TEACHER**
Provide more examples as needed.

If there are more than two addends in the problem the first step to do is to combine addends with same signs and then get the difference of their sums.
Examples:

1. \((-14) + (22) + (8) + (-16) = -(14 + 16) + (22 + 8)\)
   \[= -30 + 30 = 0\]
2. \(31 + 70 + 9 + (-155) = (31 + 70 + 9) + (-155)\)
   \[= 110 + (-155) = -45\]

**III. Exercises**

A. Who was the first English mathematician who first used the modern symbol of equality in 1557?

(To get the answer, compute the sums of the given exercises below. Write the letter of the problem corresponding to the answer found in each box at the bottom).

<table>
<thead>
<tr>
<th>A</th>
<th>25 + 95</th>
<th>C.  (30) + (-20)</th>
<th>R</th>
<th>65 + 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>38 + (-15)</td>
<td>D.  (110) + (-75)</td>
<td>O</td>
<td>(-120) + (-35)</td>
</tr>
<tr>
<td>O</td>
<td>45 + (-20)</td>
<td>T.  (16) + (-38)</td>
<td>R</td>
<td>(165) + (-85)</td>
</tr>
<tr>
<td>R</td>
<td>(-65) + (-20)</td>
<td>R.  (-65) + (-40)</td>
<td>E</td>
<td>47 + 98</td>
</tr>
<tr>
<td>E</td>
<td>(78) + (-15)</td>
<td>E.  (-75) + (20)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>-105</th>
<th>25</th>
<th>63</th>
<th>23</th>
<th>-85</th>
<th>-22</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>140</td>
<td>-55</td>
<td>10</td>
<td>-155</td>
<td>80</td>
<td>35</td>
</tr>
</tbody>
</table>
Answer: ROBERT RECORDE

B. Add the following:

1. \((18) + (-11) + (3)\)
2. \((-9) + (-19) + (-6)\)
3. \((-4) + (25) + (-15)\)
4. \((50) + (-13) + (-12)\)
5. \((-100) + (48) + (49)\)

Answers:
1. 10  2. -34  3. 6  4. 25  5. -3

C. Solve the following problems:

1. Mrs. Reyes charged PhP3,752.00 worth of groceries on her credit card. Find her balance after she made a payment of PhP2,530.00.
   Answer: PhP1,222.00

2. In a game, Team Azcals lost 5 yards in one play but gained 7 yards in the next play. What was the actual yardage gain of the team? Answer: \((-5) + 7 = 2\) yards

3. A vendor gained PhP50.00 on the first day; lost PhP28.00 on the second day, and gained PhP49.00 on the third day. How much profit did the vendor gain in 3 days? Answer: 50 + (-28) + 49 = 71. Profit is PhP71.00

4. Ronnie had PhP2280 in his checking account at the beginning of the month. He wrote checks for PhP450, PhP1200, and PhP900. He then made a deposit of PhP1000. If at any time during the month the account is overdrawn, a PhP300 service charge is deducted. What was Ronnie’s balance at the end of the month?
   Answer: \(2280 + (-450) + (-1200) + (-900) = -270\)
            \((-270) + (-300) + 1000 = 430\)
   Balance is PhP430.00

NOTE TO THE TEACHER

Summarize the two models used in this lesson. It is always good to keep these models in mind but make sure that students learn to let go of these models and should be able to add integers eventually even without these models.

Summary

In this lesson, you learned how to add integers using two different methods. The number line model is practical for small integers. For larger integers, the signed tiles model provides a more useful tool.
Lesson 4.2: Fundamental Operation on Integers: Subtraction of Integers
Time: 1 hour

Prerequisite Concepts: Whole numbers, Exponents, Concept of Integers, Addition of Integers

About the Lesson: This lesson focuses on the subtraction of integers using different approaches. It is a review of what the students learned in Grade 6.

Objectives:
In this lesson, you are expected to:
1. Subtract integers using
   a. Number line
   b. Signed tiles
2. Solve problems involving subtraction of integers.

NOTE TO THE TEACHER
This lesson is a continuation of lesson 4.1 in a sense that mastery of the law of signs in addition of integers makes subtraction easy for the learners. Emphasis must be given to how the law of signs in addition is connected to that of subtraction.

Lesson Proper:
I. Activity
Study the material below.

1. Subtraction as the reverse operation of addition.
Recall how subtraction is defined. We have previously defined subtraction as the reverse operation of addition. This means that when we ask “what is 5 minus 2?”, we are also asking “what number do we add to 2 in order to get 5?” Using this definition of subtraction, we can deduce how subtraction is done using the number line.

   a. Suppose you want to compute \((-4) - 3\). You ask “What number must be added to 3 to get \(-4\)?

      To get from 3 to \(-4\), you need to move 7 units to the left. This is equivalent to adding \(-7\) to 3. Hence in order to get \(-4\), \(-7\) must be added to 3. Therefore,

      \((-4) - 3 = -7\)

   b. Compute \((-8) - (-12)\)
What number must be added to $-12$ to get $-8$?

To go from $-12$ to $-8$, move 4 units to the right, or equivalently, add 4. Therefore,

$$(-8) - (-12) = 4$$

2. **Subtraction as the addition of the negative**

Subtraction is also defined as the addition of the negative of the number. For example, $5 - 3 = 5 + (-3)$. Keeping in mind that $n$ and $-n$ are negatives of each other, we can also have $5 - (-3) = 5 + 3$. Hence the examples above can be solved as follows:

$$(-4) - 3 = (-4) + (-3) = -7$$

$$(-8) - (-12) = (-8) + 12 = 4$$

This definition of subtraction allows the conversion of a subtraction problem to an addition problem.

**NOTE TO THE TEACHER**

You need to follow up on the opening activity, hence the problem below is important to reinforce what was discussed.

Problem:

Subtract $(-45)$ from 39 using the two definitions of subtraction.

Can you draw your number line? Where do you start numbering it to make the line shorter?

Solution:

1. $39 - (-45)$

What number must be added to $-45$ in order to obtain 39?
39 - (-45) = 84

2. 39 - (-45) = 39 + 45 = 84

II. Questions/Points to Ponder

Rule in Subtracting Integers
In subtracting integers, add the negative of the subtrahend to the minuend,
\[ a - b = a + (-b) \]
\[ a - (-b) = a + b \]

NOTE TO THE TEACHER
Give more examples as needed. The next section relies on the use of colored counters or signed tiles. You, the teacher, should study the material so that you may be able to guide your students in understanding the use of these tiles correctly.

Using signed tiles or colored counters
Signed tiles or colored counters can also be used to model subtraction of integers. In this model, the concept of subtraction as “taking away” is utilized.

Examples:

1. \( 10 - 6 \) means take away 6 from 10. Hence

   ![Diagram of 10 - 6]

   \[ 10 - 6 = 4 \]

2. \( -3 - (-2) \)

   ![Diagram of -3 - (-2)]

   \[ -3 - (-2) = -1 \]
3. $4 - 9$

Since there are not enough counters from which to take away 9, we add 9 black counters and 9 white counters. Remember that these added counters are equivalent to zero.

We now take away 9 black counters.

Notice that this configuration is the same configuration for $4 + (-9)$. We proceed with the addition and obtain the answer $-5$.

4. $2 - (-4)$

Hence $2 - (-4) = 6$

The last two examples above illustrate the definition of subtraction as the addition of the negative.

$m - n = m - n + [n + (-n)] = [m - n + n] + (-n) = m + (-n)$
III. Exercises
A. What is the name of the 4th highest mountain in the world?
(Decode the answer by finding the difference of the following subtraction problems. Write the letter to the answer corresponding to the item in the box provided below:
O Subtract (-33) from 99
L Subtract (-30) from 49
H 18 less than (-77)
E Subtract (-99) from 0
T How much is 0 decreased by (-11)?
S (-42) – (-34) – (-9) - 18

<p>| | | | | |</p>
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</tr>
</thead>
<tbody>
<tr>
<td>79</td>
<td>-95</td>
<td>132</td>
<td>11</td>
<td>-17</td>
</tr>
<tr>
<td>99</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Answer: LHOTSE

B. Mental Math
Give the difference:
1. 53 - 25
2. (-6) - 123
3. (-4) - (-9)
4. 6 - 15
5. 16 - (-20)
6. 25 - 43
7. (-30) - (-20)
8. (-19) - 2
9. 30 - (-9)
10. (-19) - (-15)

Answers:

<table>
<thead>
<tr>
<th>1. 28</th>
<th>2. -129</th>
<th>3. 5</th>
<th>4. -9</th>
<th>5. 36</th>
</tr>
</thead>
</table>

C. Solve the ff. Problems:
1. Maan deposited P53,400.00 in her account and withdrew P19,650.00 after a week. How much of her money was left in the bank?
   Answer: PhP33,750.00
2. Two trains start at the same station at the same time. Train A travels 92km/h, while train B travels 82km/h. If the two trains travel in opposite directions, how far apart will they be after an hour? If the two trains travel in the same direction, how far apart will they be in two hours?
   Answer: 92-(-82)=174 km apart
   \[2 \times 92 - 2 \times 82 = 20 \text{ km apart}\]
3. During the Christmas season. The student gov't association was able to solicit 2,356 grocery items and was able to distribute 2,198 to one barangay. If this group decided to distribute 1,201 grocery items to the next barangay, how many more grocery items did they need to solicit?
   Answer: 2356-2198=158 left after the first barangay
   \[1201-158=1043 \text{ needed for the second barangay}\]
NOTE TO THE TEACHER

To end, emphasize the new ideas that this lesson discussed, particularly the new concepts of subtraction and how these concepts allow the conversion of subtraction problems to addition problems.

Summary

In this lesson, you learned how to subtract integers by reversing the process of addition, and by converting subtraction to addition using the negative of the subtrahend.
Lesson 4.3: Fundamental Operations on Integers: Multiplication of Integers

Time: 1 hour

Prerequisite Concepts: Operations on whole numbers, addition and subtraction of integers

About the Lesson: This is the third lesson on operations on integers. The intent of the lesson is to deepen what students have learned in Grade 6, by expounding on the meaning of multiplication of integers.

Objective:
In this lesson; you are expected to:
1. Multiply integers.
2. Apply multiplication of integers in solving problems

NOTE TO THE TEACHER
The repeated addition model for multiplication can be extended to multiplication of two integers in which one of the factors is positive. However, for products in which both factors are negative, repeated addition does not have any meaning. Hence multiplication of integers will be discussed in two parts: the first part looks into products with at least one positive factor, while the second studies the product of two negative integers.

Lesson Proper:
I. Activity
Answer the following question.

How do we define multiplication?

We learned that with whole numbers, multiplication is repeated addition. For example, \(4 \times 3\) means three groups of 4. Or, putting it into a real context, 3 cars with 4 passengers each, how many passenger in all? Thus

\[4 \times 3 = 4 + 4 + 4 = 12.\]

But, if there are 4 cars with 3 passengers each, in counting the total number of passengers, the equation is \(3 \times 4 = 3 + 3 + 3 + 3 = 12\). We can say then that

\[4 \times 3 = 3 \times 4\]

and

\[4 \times 3 = 3 \times 4 = 3 + 3 + 3 + 3 = 12.\]

We extend this definition to multiplication of a negative integer by a positive integer. Consider the situation when a boy loses P6 for 3 consecutive days. His total loss for three days is

\((-6) \times 3.\) Hence, we could have

\[(-6) \times 3 = (-6) + (-6) + (-6) = -18.\]
II. Questions/Points to Ponder

The following examples illustrate further how integers are multiplied.

Example 1. Multiply : \(5 \times (-2)\)

However,

\[5 \times (-2) = (-2) \times (5)\]

Therefore:

\[(-2) \times (5) = (-2) + (-2) + (-2) + (-2) + (-2) = -10\]

The result shows that the product of a negative multiplier and a positive multiplicand is a negative integer.

**Generalization:** Multiplying unlike signs

We know that adding negative numbers means adding their positive equivalents and attaching the negative sign to the result, then

\[a \times (-b) = (-b) \times a = (-b) + (-b) + \ldots + (-b) = - (b + b + \ldots + b) = -ab\]

for any positive integers \(a\) and \(b\).

We know that any whole number multiplied by 0 gives 0. Is this true for any integer as well? The answer is YES. In fact, any number multiplied by 0 gives 0. This is known as the **Zero Property**.

**FOR THE TEACHER: PROOF OF THE ZERO PROPERTY**

Since 1 is the identity for multiplication, for any integer \(a\), \(a \times 1 = a\).

The identity for addition is 0, so \(a \times 1 = a \times (1 + 0) = a\).

By the distributive law, \(a \times (1 + 0) = a \times 1 + a \times 0 = a\).

Hence \(a + a \times 0 = a\).

Now 0 is the only number which does not change \(a\) on addition. Therefore \(a \times 0 = 0\).

What do we get when we multiply two negative integers?

Example 2. Multiply: \((-8) \times (-3)\)

We know that \((-8) \times 3 = -24\).

Therefore,

\[(-8) \times (-3) = (-8) \times 3 + (-8) \times (-3)\]
\[= (-8) \times [3 + (-3)] (\text{Distributive Law})\]
\[= (-8) \times 0 \quad (3 \text{ and } -3 \text{ are additive inverses})\]
\[= 0 \quad (\text{Zero Property})\]

The only number which when added to \(-24\) gives 0 is the additive inverse of \(-24\). Therefore, \((-8) \times (-3)\) is the additive inverse of 24, or

\[(-8) \times (-3) = 24\]

The result shows that the product of two negative integers is a positive integer.
NOTE TO THE TEACHER

The above argument can be generalized to obtain the product \((-a)\times(-b)\). The proof may be presented to more advanced students. It is important to note that the definition of the product of two negative integers is not based on the same model as the product of whole numbers (i.e., repeated addition). The basis for the definition of the product of two negative numbers is the preservation of the properties or axioms of whole number operations (distributive law, identity and inverse property).

Generalization: Multiplying Two Negative Integers
If \(a\) and \(b\) are positive integers, then \((-a) \times (-b) = ab\).

Rules in Multiplying Integers:
In multiplying integers, find the product of their positive equivalents.
1. If the integers have the same signs, their product is positive.
2. If the integers have different signs their product is negative.

III. Exercises
A. Find the product of the following:

1. \((5)(12)\)
2. \((-8)(4)\)
3. \((-5)(3)(2)\)
4. \((-7)(4)(-2)\)
5. \((3)(8)(-2)\)
6. \((9)(-8)(-9)\)
7. \((-9)(-4)(-6)\)

B. How can a person fairly divide 10 apples among 8 children so that each child has the same share.

To solve the dilemma, match the letter in column II with the number that corresponds to the numbers in column I.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((6)(-12))</td>
<td>C 270</td>
</tr>
<tr>
<td>2. ((-13)(-13))</td>
<td>P -72</td>
</tr>
<tr>
<td>3. ((19)(-17))</td>
<td>E 300</td>
</tr>
<tr>
<td>4. ((-15)(29))</td>
<td>K -323</td>
</tr>
<tr>
<td>5. ((165)(0))</td>
<td>A -435</td>
</tr>
<tr>
<td>6. ((-18)(-15))</td>
<td>M 0</td>
</tr>
<tr>
<td>7. ((-15)(-20))</td>
<td>L 16</td>
</tr>
<tr>
<td>8. ((-5)(-5)(-5))</td>
<td>J -125</td>
</tr>
<tr>
<td>9. ((-2)(-2)(-2)(-2))</td>
<td>U 169</td>
</tr>
<tr>
<td>10. ((4)(6)(8))</td>
<td>I 192</td>
</tr>
</tbody>
</table>

Answers:
1. 60  2. –32  3. –30  4. 56
5. –48  6. 648  7. –216
C. Problem Solving

1. Jof has twenty P5 coins in her coin purse. If her niece took 5 of the coins, how much has been taken away?
   **Answer:** PhP25 (5 \times 5 = 25)

2. Mark can type 45 words per minute, how many words can Mark type in 30 minutes?
   **Answer:** 1350 words (45 \times 30 = 1350)

3. Give an arithmetic equation which will solve the following
   a. The messenger came and delivered 6 checks worth PhP50 each. Are you richer or poorer? By how much?
   b. The messenger came and took away 3 checks worth PhP120 each. Are you richer or poorer? By how much?
   c. The messenger came and delivered 12 bills for PhP86 each. Are you richer or poorer? By how much?
   d. The messenger came and took away 15 bills for PhP72 each. Are you richer or poorer? By how much?

   **Answers:**
   
   | a. 6 \times 50 = 300 | Richer by PhP300 |
   | b. \(-3 \times 120 = -360 | Poorer by PhP360 |
   | c. 12 \times \(-86) = -1032 | Poorer by PhP1032 |
   | d. \(-15) \times \(-72) = 1080 | Richer by PhP1080 |

**NOTE TO THE TEACHER**

Give additional problems and drills, if only to reinforce the rules for multiplying integers. Summarize by emphasizing as well the different types of problems given in this lesson.

**Summary**

This lesson emphasized the meaning of multiplication to set the rules for multiplying integers. To multiply integers, first find the product of their positive equivalents. If the integers have the same signs, their product is positive. If the integers have different signs their product is negative.
Lesson 4.4: Fundamental Operations on Integers: Division of Integers
Time: 1 hour

Prerequisite Concepts: Addition and subtraction of Integers, Multiplication of Integers

Objective:
In this lesson you are expected to:
1. Find the quotient of two integers.
2. Solve problems involving division of integers.

NOTE TO THE TEACHER
This is a short lesson because the sign rules for division of integers are the same as with the multiplication of integers. Division is to be understood as the reverse operation of multiplication, hence making the rules the same with respect to the sign of the quotient.

Lesson Proper:
I. Activity
Answer the following questions:
What is (-51) ÷ (-3)?
What is (-51) ÷ 3?
What is 51 ÷ (-3)?
What are the rules in dividing integers?

NOTE TO THE TEACHER
This exercise emphasizes the need to remember the sign rules for dividing integers.

II. Questions/Points to Ponder
We have learned that Subtraction is the inverse operation of Addition, in the same manner, Division is the inverse operation of Multiplication.

Example 1. Find the quotient of (-51) and (-3)
Solution:
Since division is the inverse of multiplication, determine what number multiplied by (-3) produces (-51).

If we ignore the signs for the meantime, we know that
3 × 17 = 51
We also know that in order to get a negative product, the factors must have different signs. Hence (-3) × 17 = -51
Therefore
(-51) ÷ (-3) = 17

Example 2. What is (-57) ÷ 19?
Solution: 19 × 3 = 57
Hence
19 × (-3) = -57
Therefore
(-57) ÷ 19 = -3
Example 3. Show why \( 273 ÷ (-21) = -13 \).

Solution: \((-13) \times (-21) = 57\)

Therefore, \( 273 ÷ (-21) = -13 \)

NOTE TO THE TEACHER
It is important to give more examples to students. Always, ask students to explain or justify their answers.

Generalization
The quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer. However, division by zero is not possible.

NOTE TO THE TEACHER
Since we introduced division as the reverse operation of multiplication, it is now easy to show why division by 0 is not possible.

Example 4. Perform the indicated operations

1. \( 2 - 3 \times (-4) \)
2. \( 4 \times 5 + 72 ÷ (-6) \)
3. \( 9 + 6 - (-3) \times 12 ÷ (-9) \)

Solution:
1. \( 2 - 3 \times (-4) = 2 - (-12) = 14 \)
2. \( 4 \times 5 + 72 ÷ (-6) = 20 + (-12) = 8 \)
3. \( 9 + 6 - (-3) \times 12 ÷ (-9) = 9 + 6 - (-36) ÷ (-9) = 9 + 6 - 4 = 11 \)

III. Exercises:
A. Compute the following

\[
\begin{align*}
1. \quad & (10 + 15) - 4 \times 3 + 7 \times (-2) \\
2. \quad & 22 \times 9 ÷ (-6) - 5 \times 8 \\
3. \quad & 36 ÷ 12 + 53 ÷ (-30) \\
4. \quad & (30 + 26) ÷ [(-2) \times 7] \\
5. \quad & (124 - 5 \times 12) ÷ 8
\end{align*}
\]

Answers:
1. \(-1\) 2. \(-73\) 3. \(26\) 4. \(-4\) 5. \(8\)

B. What was the original name for the butterfly?

To find the answer find the quotient of each of the following and write the letter of the problems in the box corresponding to the quotient.
C. Solve the following problems:

1. Vergara’s store earned P8750 a week. How much is her average earning in a day? \textbf{Answer:} PhP1250.00 \((8750 \div 7 = 1250)\)

2. Russ worked in a factory and earned P7875.00 for 15 days. How much is his earning in a day? \textbf{Answer:} PhP525.00 \((7875 \div 15 = 525)\)

3. There are 336 oranges in 12 baskets. How many oranges are there in 3 baskets? \textbf{Answer:} 84 oranges \((336 \div 12 \times 3 = 84)\)

4. A teacher has to divide 280 pieces of graphing paper equally among his 35 students. How many pieces of graphing paper will each student receive? \textbf{Answer:} 8 \((280 \div 35 = 8)\)

5. A father has 976 sq. meters lot, he has to divide it among his 4 children. What is the share of each child? \textbf{Answer:} 244 sq. meters \((976 \div 4 = 244)\)

D. Complete the three-by-three magic square (that is, the sums of the numbers in each row, in each column and in each of the diagonals are the same) using the numbers \(-10, -7, -4, -3, 0, 3, 4, 7, 10\). What is the sum for each row, column and diagonal?

\[
\begin{array}{ccc}
R & -352 \div & U \\
(128) \div & L & (-444) \div (-12) \\
Y & (144) \div -3 & E \\
(-147) \div 7 & F & (-315) \div (-) \\
\end{array}
\]
Answer: The sum of all the numbers is 0. Hence each column/row/diagonal will have a sum of $0 \div 3 = 0$. Put 0 in the middle square. Put each number and its negative on either side of 0. A possible solution is

\[
\begin{array}{ccc}
7 & 10 & 3 \\
-4 & 0 & 4 \\
-3 & -10 & -7 \\
\end{array}
\]

Summary
Division is the reverse operation of multiplication. Using this definition, it is easy to see that the quotient of two integers with the same signs is a positive integer and the quotient of two integers having unlike signs is a negative integer.
Lesson 5: Properties of the Operations on Integers

Time: 1.5 hours

Prerequisite Concepts: Addition, Subtraction, Multiplication and Division of Integers

Objectives
In this lesson, you are expected to:

1. State and illustrate the different properties of the operations on integers
   a. closure
   b. commutative
   c. associative
   d. distributive
   e. identity
   f. inverse

2. Rewrite given expressions according to the given property.

NOTE TO THE TEACHER:
Operations on integers are some of the difficult topics in elementary algebra and one of the least mastered skills of students based on researches. The different activities presented in this lesson will hopefully give the students a tool for creating their own procedures in solving equations involving operations on integers. These are the basic rules of our system of algebra and they will be used in all succeeding mathematics. It is very important that students understand how to apply each property when solving math problems.

In activities 1 and 2, the teacher will try to test the students’ ability to give corresponding meaning to the different words exhibited and later on relate said terms to the lesson. In addition, students can show some creativity in activity 2.

Lesson Proper:
I. A. Activity 1: Try to reflect on these . . .
   1. Give at least 5 words synonymous to the word “property”.

Activity 2: PICTONARY GAME: DRAW AND TELL!
The following questions will be answered as you go along to the next activity.

- What properties of real numbers were shown in the Pictionary Game?
  
  Give one example and explain.
  
- How are said properties seen in real life?

**NOTE TO THE TEACHER**

Activity 3 gives a visual presentation of the properties.

**Activity 3: SHOW AND TELL!**

Determine what kind of property of real numbers is being illustrated in the following images:

A. Fill in the blanks with the correct numerical values of the motorbike and bicycle riders.

\[ \underline{\text{number of motorbike riders}} + \underline{\text{number of bicycle riders}} = \underline{\text{number of motorbike riders}} + \underline{\text{number of bicycle riders}} \]

If \( a \) represents the number of motorbike riders and \( b \) represents the number of bicycle riders, show the mathematical statement for the diagram below.

\[ \underline{\text{number of motorbike riders}} + \underline{\text{number of bicycle riders}} = \underline{\text{number of motorbike riders}} + \underline{\text{number of bicycle riders}} \]

*Expected Answer: \( a + b = b + a \)*

**Guide Questions:**

- What operation is used in illustrating the diagram? **Addition**
- What happened to the terms in both sides of the equation? **The terms were interchanged.**
- Based on the previous activity, what property is being applied?
Commutative Property of Addition: For integers $a, b$, $a + b = b + a$

- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
  
  $2 \cdot 3 = 3 \cdot 2$
  
  $6 = 6$

Commutative Property of Multiplication: For integers $a, b$, $ab = ba$

- Define the property.

**Commutative Property**
Changing the order of two numbers that are either being added or multiplied does not change the result.

- Give a real life situation in which the commutative property can be applied.
  
  *An example is preparing fruit juices - even if you put the powder first before the water or vice versa, the product will still be the same. It's still the same fruit juice.*

- Test the property on subtraction and division operations by using simple examples. What did you discover?
  
  Commutative property is not applicable to subtraction and division as shown in the following examples:

  $6 - 2 = 2 - 6$
  
  $4 \neq -4$

  $6 \div 2 = 2 \div 6$
  
  $3 \neq$

B. Fill in the blanks with the correct numerical values of the set of cellphones, ipods and laptops.

```
+ + +
= 
```
If \(a\) represents the number of cellphones, \(b\) represents the ipods and \(c\) represents the laptops, show the mathematical statement for the diagram below.

\[
(\underline{_______} + \underline{_______}) + \underline{_______} = \underline{_______} + (\underline{_______} + \underline{_______})
\]

Expected Answer: \((a + b) + c = a + (b + c)\)

**Guide Questions:**

What operation is used in illustrating the diagram? **Addition**
- What happened to the groupings of the given sets that correspond to both sides of the equation? **The groupings were changed.**
- Based on the previous activity, what property is being applied? **Associative Property of Addition**
  - For integers \(a, b\) and \(c\), \((a + b) + c = a + (b + c)\)
- What if the operation is replaced by multiplication, will the same property be applicable? Give an example to prove your answer.
  
  \[
  (2 \cdot 3) \cdot 5 = 3 \cdot (2 \cdot 5) \\
  6 \cdot 5 = 3 \cdot 10 \\
  30 = 30
  \]

**Associative Property of Multiplication**
- For integers \(a, b\) and \(c\), \((a \cdot b)c = a(b \cdot c)\)
- Define the property. **Associative Property**  
  Changing the grouping of numbers that are either being added or multiplied does not change its value.
- Give a real life situation wherein associative property can be applied. An example is preparing instant coffee – even if you combine coffee and creamer then sugar or coffee and sugar then creamer the result will be the same – 3-in-1 coffee.
- Test the property on subtraction and division operations by using simple examples. What did you discover? **Associative property is not applicable to subtraction and division as shown in the following examples:**

\[
(6 - 2) - 1 = 6 - (2 - 1) \\
(12 ÷ 2) ÷ 2 = 12 ÷ (2 ÷ 2) \\
4 - 1 = 6 - 1 \\
6 ÷ 2 = 12 ÷ 1 \\
3 \neq 5 \\
3 \neq 12
\]
C. Fill in the blanks with the correct numerical values of the set of oranges and set of strawberries.

\[
\begin{array}{c}
2 \times \left\{ \begin{array}{c}
\text{oranges}
\end{array} \right\} + \left\{ \begin{array}{c}
\text{strawberries}
\end{array} \right\}
\end{array}
\]

equals

\[
\begin{array}{c}
2 \times \left\{ \begin{array}{c}
\text{oranges}
\end{array} \right\} + \left\{ \begin{array}{c}
\text{strawberries}
\end{array} \right\}
\end{array}
\]

If \( a \) represents the multiplier in front, \( b \) represents the set of oranges and \( c \) represents the set of strawberries, show the mathematical statement for the diagram below.

\[
_____ (______ + _______ ) = _______ \cdot _______ + _______ \cdot _______
\]

Answer: \( a(b + c) = ab + ac \)

Guide Questions:
- Based on the previous activity, what property is being applied in the images presented?
  Distributive Property
  For any integers \( a, b, c \), \( a(b + c) = ab + ac \)
  For any integers \( a, b, c \), \( a(b - c) = ab - ac \)
- Define the property.
  Distributive Property
  When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same
when each number is multiplied by the factor and the products are then added / subtracted.

- In the said property can we add/subtract the numbers inside the parentheses and then multiply or perform multiplication first and then addition/subtraction? Give an example to prove your answer.

In the example, we can either add or subtract the numbers inside the parentheses first and then multiply the result; or, we can multiply with each term separately and then add/ subtract the two products together. The answer is the same in both cases as shown below.

\[-2(4 + 3) = (-2 \cdot 4) + (-2 \cdot 3)\]
\[-2(7) = (-8) + (-6)\]
\[-14 = -14\]

or

\[-2(4 + 3) = -2(7)\]
\[-2(7) = -14\]
\[-14 = -14\]

- Give a real life situation wherein distributive property can be applied.
  
Your mother gave you four 5-peso coins and your grandmother gave you four 20-peso bills. You now have PhP20 worth of 5-peso coins and PhP80 worth of 20-peso bill. You also have four sets of PhP25 each consisting of a 5-peso coin and a 20-peso bill.

D. Fill in the blanks with the correct numerical representation of the given illustration.

\[
\begin{array}{c}
\text{Answer: } a + 0 = a
\end{array}
\]

**Guide Questions:**
- Based on the previous activity, what property is being applied in the images presented?
  
  Identity Property for Addition
  
  \[a + 0 = a\]
• What will be the result if you add something represented by any number to nothing represented by zero? **The result is the non-zero number.**

• What do you call zero “0” in this case? **Zero, “0” is the additive identity.**

• Define the property. 
  **Identity Property for Addition** states that 0 is the additive identity, that is, the sum of any number and 0 is the given number.

• Is there a number multiplied to any number that will result to that same number? Give examples.
  Yes, the number is 1.
  Examples: \(1 \cdot 2 = 2\) \(1 \cdot 3 = 2\) \(1 \cdot 4 = 2\)

• What property is being illustrated? Define.
  **Identity Property for Multiplication** says that 1 is the Multiplicative Identity
  - the product of any number and 1 is the given number, \(a \cdot 1 = a\).

• What do you call one “1” in this case?
  **One, “1” is the multiplicative identity**

E. Give the correct mathematical statement of the given illustrations. To do this, refer to the guide questions below.

---

**Guide Questions:**
- How many cabbages are there in the crate? 14 cabbages

---
• Using integers, represent “put in 14 cabbages” and “remove 14 cabbages”? What will be the result if you add these representations? (+14) + (-14) = 0

• Based on the previous activity, what property is being applied in the images presented? 
  **Inverse Property for Addition**
  \[ a + (-a) = 0 \]

• What will be the result if you add something to its negative? **The result is always zero.**

• What do you call the opposite of a number in terms of sign? What is the opposite of a number represented by \( a \)?
  **Additive Inverse.** The additive inverse of the number \( a \) is \(-a\).

• Define the property.
  **Inverse Property for Addition**
  - states that the sum of any number and its additive inverse or its negative, is zero.

• What do you mean by reciprocal and what is the other term used for it?
  The reciprocal is **1 divided by that number** or the fraction \( \frac{1}{a} \), where \( a \) represents the number.
  The reciprocal of a number is also known as its multiplicative inverse.

• What if you multiply a number say 5 by its multiplicative inverse \( \frac{1}{5} \), what will be the result? \( 5 \cdot \frac{1}{5} = 1 \)

• What property is being illustrated? Define.
  **Inverse Property for Multiplication**
  - states that the product of any number and its multiplicative inverse or reciprocal, is 1.

  **For any number \( a \), the multiplicative inverse is** \( \frac{1}{a} \).

*Important Terms to Remember*

The following are terms that you must remember from this point on.

1. **Closure Property**
   Two integers that are added and multiplied remain as integers. The set of integers is closed under addition and multiplication.

2. **Commutative Property**
   Changing the order of two numbers that are either being added or multiplied does not change the value.

3. **Associative Property**
   Changing the grouping of numbers that are either being added or multiplied does not change its value.

4. **Distributive Property**
When two numbers have been added / subtracted and then multiplied by a factor, the result will be the same when each number is multiplied by the factor and the products are then added / subtracted.

5. Identity Property
   - **Additive Identity**
     - states that the sum of any number and 0 is the given number. Zero, “0” is the additive identity.
   - **Multiplicative Identity**
     - states that the product of any number and 1 is the given number, \( a \cdot 1 = a \). One, “1” is the multiplicative identity.

6. Inverse Property
   - **In Addition**
     - states that the sum of any number and its additive inverse, is zero.
     - The additive inverse of the number \( a \) is \(-a\).
   - **In Multiplication**
     - states that the product of any number and its multiplicative inverse or reciprocal, is 1. The multiplicative inverse of the number \( a \) is \( \frac{1}{a} \).

**Notations and Symbols**

In this segment, you will learn some of the notations and symbols pertaining to properties of real number applied in the operations of integers.

<table>
<thead>
<tr>
<th>Closure Property under addition and multiplication</th>
<th>( a, b \in I ), then ( a+b \in I ), ( a\cdot b \in I )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative property of addition</td>
<td>( a + b = b + a )</td>
</tr>
<tr>
<td>Commutative property of multiplication</td>
<td>( ab = ba )</td>
</tr>
<tr>
<td>Associative property of addition</td>
<td>( (a + b) + c = a + (b + c) )</td>
</tr>
<tr>
<td>Associative property of multiplication</td>
<td>( (ab) c = a (bc) )</td>
</tr>
<tr>
<td>Distributive property</td>
<td>( a(b + c) = ab + ac )</td>
</tr>
<tr>
<td>Additive identity property</td>
<td>( a + 0 = a )</td>
</tr>
<tr>
<td>Multiplicative identity property</td>
<td>( a \cdot 1 = a )</td>
</tr>
<tr>
<td>Multiplicative inverse property</td>
<td>( \frac{1}{a} \cdot a = 1 )</td>
</tr>
<tr>
<td>Additive inverse property</td>
<td>( a + (-a) = 0 )</td>
</tr>
</tbody>
</table>
NOTE TO THE TEACHER:

It is important for you to examine and discuss the responses by your students to the questions posed in every activity and exercise in order to practice what they have learned for themselves. Remember application as part of the learning process is essential to find out whether the learner gained knowledge of the concept or not. It is also appropriate to encourage brainstorming, dialogues and arguments in the class. After the exchanges, see to it that all questions are resolved.

III. Exercises

A. Complete the Table: Which property of real number justifies each statement?

<table>
<thead>
<tr>
<th>Given</th>
<th>Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0 + (-3) = -3</td>
<td>Additive Identity Property</td>
</tr>
<tr>
<td>2. 2(3 - 5) = 2(3) - 2(5)</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>3. (-6) + (-7) = (-7) + (-6)</td>
<td>Commutative Property</td>
</tr>
<tr>
<td>4. 1 x (-9) = -9</td>
<td>Multiplicative Identity Property</td>
</tr>
<tr>
<td>5. -4 x 1/4 = 1</td>
<td>Multiplicative Inverse Property</td>
</tr>
<tr>
<td>6. 2 x (3 x 7) = (2 x 3) x 7</td>
<td>Associative Property</td>
</tr>
<tr>
<td>7. 10 + (-10) = 0</td>
<td>Additive Inverse Property</td>
</tr>
<tr>
<td>8. 2(5) = 5(2)</td>
<td>Commutative Property</td>
</tr>
<tr>
<td>9. 1 x (-3/4) = -3/4</td>
<td>Multiplicative Identity Property</td>
</tr>
<tr>
<td>10. (-3)(4 + 9) = (-3)(4) + (-3)(9)</td>
<td>Distributive Property</td>
</tr>
</tbody>
</table>

B. Rewrite the following expressions using the given property.

1. 12a - 5a = (12-5)a
2. (7a)b = 7(ab)
3. 8 + 5 = 5 + 8
4. -4(1) = -4
5. 25 + (-25) = 0

C. Fill in the blanks and determine what properties were used to solve the equations.

1. 5 x (-2 + 2) = 0 Additive Inverse, Zero Property
2. -4 + 4 = 0 Additive Inverse
3. -6 + 0 = -6 Additive Identity
4. (-14 + 14) + 7 = 7 Additive Inverse, Additive Identity
5. 7 x (0 + 7) = 49 Additive Identity
NOTE TO THE TEACHER
Try to give more of the type of exercises in Exercise C. Combine properties so that you can test how well your students have understood the lesson.

Summary
The lesson on the properties of real numbers explains how numbers or values are arranged or related in an equation. It further clarifies that no matter how these numbers are arranged and what processes are used, the composition of the equation and the final answer will still be the same. Our society is much like these equations - composed of different numbers and operations, different people with varied personalities, perspectives and experiences. We can choose to look at the differences and forever highlight one’s advantage or superiority over the others. Or we can focus on the commonality among people and altogether, work for the common good. A peaceful society and harmonious relationship starts with recognizing, appreciating and fully maximizing the positive traits that we, as a people, have in common.
Lesson 6: Rational Numbers in the Number Line
Time: 1 hour

Prerequisite Concepts: Subsets of Real Numbers, Integers

Objective:
In this lesson, you, the students, are expected to
1. Define rational numbers;
2. Illustrate rational numbers on the number line;
3. Arrange rational numbers on the number line.

NOTE TO THE TEACHER:
Ask students to recall the relationship of the set of rational numbers to the set of integers and the set of non-integers (Lesson 4). This lesson gives students a challenge in their numerical estimation skills. How accurately can they locate rational numbers between two integers, perhaps, or between any two numbers?

Lesson Proper
I. Activity

Determine whether the following numbers are rational numbers or not.

- 2, π, \(\frac{1}{11}\), \(\frac{\sqrt{4}}{\sqrt{16}}\), -1.89,

Now, try to locate them on the real number line below by plotting:

NOTE TO THE TEACHER:
Give as many rational numbers as class time can allow. Give them in different forms: integers, fractions, mixed numbers, decimals, repeating decimals, etc.

II. Questions to Ponder

Consider the following examples and answer the questions that follow:

a. \(7 \div 2 = 3 \frac{1}{2}\)

b. \((-25) \div 4 = -6 \frac{1}{4}\)

c. \((-6) \div (-12) = \frac{1}{2}\)

1. Are quotients integers? **Not all the time. Consider \(\frac{61}{5}\).**
2. What kind of numbers are they? **Quotients are rational numbers.**
3. Can you represent them on a number line? **Yes. Rational numbers are real numbers and therefore, they are found in the real number line.**
Recall what rational numbers are...

3 ½, -6 ¼, ½, are rational numbers. The word rational is derived from the word "ratio" which means quotient. Rational numbers are numbers which can be written as a quotient of two integers, \( \frac{a}{b} \) where \( b \neq 0 \).

The following are more examples of rational numbers:

\[
5 = \frac{5}{1} \quad 0.06 = \frac{6}{100} \quad 1.3 = \frac{13}{10}
\]

From the example, we can see that an integer is also a rational number and therefore, integers are a subset of rational numbers. Why is that?

Let's check on your work earlier. Among the numbers given, - 2, \( \pi \), \( \frac{1}{11} \), \( \sqrt[4]{4} \), \( \sqrt{16} \), - 1.89, the numbers \( \pi \) and \( \sqrt[4]{4} \) are the only ones that are not rational numbers. Neither can be expressed as a quotient of two integers. However, we can express the remaining ones as a quotient of two integers:

\[
-2 = \frac{-2}{1}, \quad \sqrt{16} = 4 = \frac{4}{1}, \quad -1.89 = \frac{-189}{100}
\]

Of course, \( \frac{1}{11} \) is already a quotient by itself.

We can locate rational numbers on the real number line.

**Example 1.** Locate ½ on the number line.

a. Since 0 < ½ < 1, plot 0 and 1 on the number line.

b. Get the midpoint of the segment from 0 to 1. The midpoint now corresponds to ½

**Example 2.** Locate 1.75 on the number line.

a. The number 1.75 can be written as \( \frac{7}{4} \) and, \( 1 < \frac{7}{4} < 2 \). Divide the segment from 0 to 2 into 8 equal parts.

b. The 7th mark from 0 is the point 1.75.
Example 3. Locate the point $-\frac{5}{3}$ on the number line.

Note that $-2 < -\frac{5}{3} < -1$. Dividing the segment from $-2$ to $0$ into $6$ equal parts, it is easy to plot $-\frac{5}{3}$. The number $-\frac{5}{3}$ is the $5$th mark from $0$ to the left.

Go back to the opening activity. You were asked to locate the rational numbers and plot them on the real number line. Before doing that, it is useful to arrange them in order from least to greatest. To do this, express all numbers in the same form — either as similar fractions or as decimals. Because integers are easy to locate, they need not take any other form. It is easy to see that

$$-2 < -1.89 < \frac{1}{11} < \sqrt{16}$$

Can you explain why?

Therefore, plotting them by approximating their location gives

### III. Exercises

1. Locate and plot the following on a number line (use only one number line).

<table>
<thead>
<tr>
<th>a. $-\frac{10}{3}$</th>
<th>e. $-0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. $2.07$</td>
<td>f. $\frac{71}{9}$</td>
</tr>
<tr>
<td>c. $\frac{2}{5}$</td>
<td>g. $0$</td>
</tr>
<tr>
<td>d. $12$</td>
<td>h. $\frac{1}{6}$</td>
</tr>
</tbody>
</table>

2. Name $10$ rational numbers that are greater than $-1$ but less than $1$ and arrange them from least to greatest on the real number line?

**Examples are:**

$\frac{1}{10}, \frac{3}{10}, \frac{1}{2}, \frac{1}{5}, \frac{1}{100}, 0, \frac{1}{8}, \frac{2}{11}, \frac{8}{37}, \frac{9}{10}$
3. Name one rational number $x$ that satisfies the descriptions below:

a. $-10 \leq x < -9$
   Possible answers:
   \[ x = -\frac{46}{5}, -\frac{48}{5}, -9.75, -9\frac{8}{9}, -9.99 \]

b. $\frac{1}{10} < x < \frac{1}{2}$
   Possible answers:
   \[ x = -\frac{46}{5}, -\frac{48}{5}, -9.75, -9\frac{8}{9}, -9.99 \]

c. $3 < x < \pi$
   Possible answers:
   \[ x = 3.1, 3.01, 3.001, 3.12 \]

d. $\frac{1}{4} < x < \frac{1}{3}$
   Possible answers:
   \[ x = \frac{13}{50}, 0.27, 0.28, \frac{299}{1000}, \frac{3}{10} \]

e. $-\frac{8}{9} < x < -\frac{1}{9}$
   Possible answers:
   \[ x = -\frac{3}{25}, -0.124, -\frac{17}{144}, -0.112 \]

NOTE TO THE TEACHER:

End this lesson with a summary as well as a preview to what students will be expecting to learn about rational numbers, their properties, operations and uses.

Summary

In this lesson, you learned more about what rational numbers are and where they can be found in the real number line. By changing all rational numbers to equivalent forms, it is easy to arrange them in order, from least to greatest or vice versa.
Lesson 7: Forms of Rational Numbers and Addition and Subtraction of Rational Numbers

Time: 2 hours

Prerequisite Concepts: definition of rational numbers, subsets of real numbers, fractions, decimals

Objectives:
In this lesson, you are expected to:
1. Express rational numbers from fraction form to decimal form (terminating and repeating and non-terminating) and vice versa;
2. Add and subtract rational numbers;
3. Solve problems involving addition and subtraction of rational numbers.

NOTE TO THE TEACHER:
The first part of this module is a lesson on changing rational numbers from one form to another, paying particular attention to changing rational numbers in non-terminating and repeating decimal form to fraction form. It is assumed that students know decimal fractions and how to operate on fractions and decimals.

Lesson Proper:
A. Forms of Rational Numbers
   I. Activity
   1. Change the following rational numbers in fraction form or mixed number form to decimal form:
      a. $\frac{1}{4} = -0.25$
      b. $\frac{3}{10} = 0.3$
      c. $\frac{5}{100} = 3.05$
      d. $\frac{5}{2} = 2.5$
      e. $\frac{17}{10} = -1.7$
      f. $\frac{-2}{5} = -2.2$

   NOTE TO THE TEACHER:
   These should be treated as review exercises. There is no need to spend too much time on reviewing the concepts and algorithms involved here.

   2. Change the following rational numbers in decimal form to fraction form.
      a. $1.8 = \frac{9}{5}$
      b. $-3.5 = \frac{-7}{2}$
      c. $-2.2 = \frac{-11}{5}$
      d. $-0.001 = \frac{-1}{1000}$
      e. $10.999 = \frac{10999}{1000}$
      f. $0.1\overline{1} = \frac{1}{9}$

   NOTE TO THE TEACHER:
The discussion that follows assumes that students remember why certain fractions are easily converted to decimals. It is not so easy to
change fractions to decimals if they are not decimal fractions. Be aware of
the fact that this is the time when the concept of a fraction becomes very
different. The fraction that students remember as indicating a part of a
whole or of a set is now a number (rational) whose parts (numerator and
denominator) can be treated separately and can even be divided! This is a
major shift in concept and students have to be prepared to understand how
these concepts are consistent with what they know from elementary level
mathematics.

II. Discussion

Non-decimal Fractions

There is no doubt that most of the above exercises were easy for you. This is
because all except item 2f are what we call decimal fractions. These numbers are all
parts of powers of 10. For example, \(- \frac{1}{4} = \frac{25}{100}\) which is easily convertible to a
decimal form, 0.25. Likewise, the number \(-3.5 = \frac{-35}{10} = \frac{-35}{10}\).

What do you do when the rational number is not a decimal fraction? How do
you convert from one form to the other?

Remember that a rational number is a quotient of 2 integers. To change a
rational number in fraction form, you need only to divide the numerator by the
denominator.

Consider the number \(-\frac{1}{8}\). The smallest power of 10 that is divisible by 8 is
1000. But, \(-\frac{1}{8}\) means you are dividing 1 whole unit into 8 equal parts. Therefore,
divide 1 whole unit first into 1000 equal parts and then take \(-\frac{1}{8}\) of the thousandths
part. That is equal to \(-\frac{125}{1000}\) or 0.125.

Example: Change \(-\frac{1}{16}, \frac{9}{11}\) and \(-\frac{1}{3}\) to their decimal forms.

The smallest power of 10 that is divisible by 16 is 10,000. Divide 1 whole unit
into 10,000 equal parts and take \(-\frac{1}{16}\) of the ten thousandths part. That is equal to
\(-\frac{625}{10000}\) or 0.625. You can obtain the same value if you perform the long division
\(1 ÷ 16\).
Do the same for $\frac{9}{11}$. Perform the long division $9 \div 11$ and you should obtain $0.\overline{81}$. Therefore, $\frac{9}{11} = 0.\overline{81}$. Also, $-\frac{1}{3} = -0.\overline{3}$. Note that both $\frac{9}{11}$ and $-\frac{1}{3}$ are non-terminating but repeating decimals.

To change rational numbers in decimal forms, express the decimal part of the numbers as a fractional part of a power of 10. For example, $-2.713$ can be changed initially to $-2\frac{713}{1000}$ and then changed to $\frac{-2173}{1000}$.

What about non-terminating but repeating decimal forms? How can they be changed to fraction form? Study the following examples:

**Example 1**: Change $0.\overline{2}$ to its fraction form.

Solution: Let $r = 0.222...$

$10r = 2.222...$

Then subtract the first equation from the second equation and obtain

$9r = 2.0$

$r = \frac{2}{9}$

Therefore, $0.\overline{2} = \frac{2}{9}$.

**Example 2**: Change $-1.\overline{35}$ to its fraction form.

Solution: Let $r = -1.353535...$

$100r = -135.353535...$

Then subtract the first equation from the second equation and obtain

$99r = -134$

$r = -\frac{134}{99} = -\frac{135}{99}$

Therefore, $-1.\overline{35} = -\frac{135}{99}$.

**NOTE TO THE TEACHER:**

Now that students are clear about how to change rational numbers from one form to another, they can proceed to learning how to add and subtract them. Students will realize soon that these skills are the same skills they learned back in elementary mathematics.
B. Addition and Subtraction of Rational Numbers in Fraction Form

I. Activity
Recall that we added and subtracted whole numbers by using the number line or by using objects in a set.

Using linear or area models, find the sum or difference.

a. \( \frac{3}{5} + \frac{1}{5} = \) _____

b. \( \frac{1}{8} + \frac{5}{8} = \) _____

c. \( \frac{10}{11} - \frac{3}{11} = \) _____

d. \( 3\frac{6}{7} - 1\frac{2}{7} = \) _____

Without using models, how would you get the sum or difference?

Consider the following examples:

1. \( \frac{1}{6} + \frac{1}{2} = \frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3} \)

2. \( \frac{6}{7} + \left( -\frac{2}{3} \right) = \frac{18}{21} + \left( -\frac{14}{21} \right) = \frac{4}{21} \)

3. \( -\frac{4}{3} + \left( -\frac{1}{5} \right) = -\frac{20}{15} + \left( -\frac{3}{15} \right) = -\frac{23}{15} \text{ or } -1\frac{8}{15} \)

4. \( \frac{14}{5} - \frac{4}{7} = \frac{98}{35} - \frac{20}{35} = \frac{78}{35} = \frac{2}{3} \)

5. \( -\frac{7}{12} - \left( -\frac{2}{3} \right) = -\frac{7}{12} - \left( -\frac{8}{12} \right) = -\frac{7 + 8}{12} = -\frac{1}{12} \)

6. \( -\frac{1}{6} - \left( -\frac{11}{20} \right) = -\frac{10}{60} - \left( -\frac{33}{60} \right) = -\frac{10 + 33}{60} = \frac{23}{60} \)

Answer the following questions:
1. Is the common denominator always the same as one of the denominators of the given fractions?
2. Is the common denominator always the greater of the two denominators?
3. What is the least common denominator of the fractions in each example?
4. Is the resulting sum or difference the same when a pair of dissimilar fractions is replaced by any pair of similar fractions?

Problem: Copy and complete the fraction magic square. The sum in each row, column, and diagonal must be 2.

\[
\begin{array}{ccc}
\text{a} & \frac{1}{2} & \text{b} \\
\frac{7}{5} & \frac{1}{3} & \text{c} \\
d & e & \frac{2}{5} \\
\end{array}
\]

» What are the values of a, b, c, d and e? \( a = \frac{1}{6}, \ b = \frac{4}{3}, \ c = \frac{4}{15}, \ d = \frac{13}{30}, \ e = \frac{7}{6} \)
Important things to remember
To Add or Subtract Fraction

- With the same denominator,

If \( a, b \) and \( c \) denote integers, and \( b \neq 0 \), then

\[
\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}
\]

- With different denominators, \( \frac{a}{b} \) and \( \frac{c}{d} \), where \( b \neq 0 \) and \( d \neq 0 \)

If the fractions to be added or subtracted are dissimilar

» Rename the fractions to make them similar whose denominator is the least common multiple of \( b \) and \( d \).

» Add or subtract the numerators of the resulting fractions.

» Write the result as a fraction whose numerator is the sum or difference of the numerators and whose denominator is the least common multiple of \( b \) and \( d \).

Examples:

To Add:                                                          To Subtract:

a. \( \frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7} \) a. \( \frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7} \)

b. \( \frac{2}{5} + \frac{1}{4} \) b. \( \frac{4}{5} - \frac{1}{4} \)

LCM/LCD of 5 and 4 is 20

\[
\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}
\]

\[
\frac{4}{5} - \frac{1}{4} = \frac{16}{20} - \frac{5}{20} = \frac{16-5}{20} = \frac{11}{20}
\]

II. Questions to Ponder (Post –Activity Discussion)

Let us answer the questions posed in activity.
You were asked to find the sum or difference of the given fractions.

a. \( \frac{3}{5} + \frac{1}{5} = \frac{4}{5} \) c. \( \frac{10}{11} - \frac{3}{11} = \frac{7}{11} \)

b. \( \frac{1}{8} + \frac{5}{8} = \frac{6}{8} \quad \text{or} \quad \frac{3}{4} \) d. \( 3\frac{6}{7} - 1\frac{2}{7} = 2\frac{4}{7} \)

Without using the models, how would you get the sum or difference?
You would have to apply the rule for adding or subtracting similar fractions.
1. Is the common denominator always the same as one of the denominators of the given fractions?

*Not always. Consider \( \frac{2}{5} + \frac{3}{4} \). Their least common denominator is 20 not 5 or 4.*

2. Is the common denominator always the greater of the two denominators?

*Not always. The least common denominator is always greater than or equal to one of the two denominators and it may not be the greater of the two denominators.*

3. What is the least common denominator of the fractions in each example?

   (1) 6  (2) 21  (3) 15  (4) 35  (5) 12  (6) 60

4. Is the resulting sum or difference the same as when a pair of dissimilar fractions is replaced by any pair of similar fractions?

*Yes, for as long as the replacement fractions are equivalent to the original fractions.*

### NOTE TO THE TEACHER:

Answers in simplest form or lowest terms could mean both mixed numbers with the fractional part in simplest form or an improper fraction whose numerator and denominator have no common factor except 1. Both are acceptable as simplest forms.

### III. Exercises

Do the following exercises.

a. Perform the indicated operations and express your answer in simplest form.

1. \( \frac{2}{9} + \frac{3}{9} + \frac{1}{9} = \frac{2}{3} \)

2. \( \frac{6}{5} + \frac{3}{5} + \frac{4}{5} = \frac{13}{5} \)

3. \( \frac{2}{5} + \frac{7}{10} = \frac{11}{10} = \frac{1}{\frac{1}{10}} \)

4. \( \frac{16}{24} - \frac{6}{12} = \frac{1}{6} \)

5. \( 2\frac{5}{12} - \frac{2}{3} = \frac{7}{4} \)

6. \( 8\frac{1}{4} + \frac{2}{7} = \frac{239}{28} = 8\frac{15}{28} \)

7. \( 3\frac{1}{4} + 6\frac{2}{3} = 9\frac{11}{12} \)

8. \( 9\frac{5}{7} - 3\frac{2}{7} = 6\frac{3}{7} \)

9. \( \frac{7}{9} - \frac{1}{12} = \frac{25}{36} \)

10. \( 11\frac{5}{9} - 7\frac{5}{6} = \frac{67}{18} = 3\frac{13}{18} \)

11. \( \frac{1}{4} + \frac{2}{3} - \frac{1}{2} = \frac{5}{12} \)

12. \( 10 - 3\frac{5}{11} = \frac{72}{11} = 6\frac{6}{11} \)

13. \( \frac{7}{20} + \frac{3}{8} + \frac{2}{5} = \frac{9}{8} \)

14. \( \frac{5}{12} + \frac{4}{9} - \frac{1}{4} = \frac{11}{18} \)

15. \( 2\frac{5}{8} + \frac{1}{2} + 7\frac{3}{4} = \frac{87}{8} = 10\frac{7}{8} \)
b. Give the number asked for.

1. What is three more than three and one-fourth? \[ \frac{1}{4} \]

2. Subtract from 15 \( \frac{1}{2} \) the sum of 2 \( \frac{1}{3} \) and 4 \( \frac{2}{5} \). What is the result? \[ \frac{263}{30} = \frac{23}{30} \]

3. Increase the sum of 6 \( \frac{3}{14} \) and 2 \( \frac{2}{7} \) by 3 \( \frac{1}{2} \). What is the result? \[ 12 \]

4. Decrease 21 \( \frac{3}{8} \) by 5 \( \frac{1}{5} \). What is the result? \[ \frac{647}{40} = \frac{167}{40} \]

5. What is \( -8 \frac{4}{5} \) minus 3 \( \frac{2}{7} \)? \[ -\frac{423}{35} = -12 \frac{3}{35} \]

NOTE TO THE TEACHER:

You should give more exercises if needed. You, the teacher should probably use the calculator to avoid computing mistakes.

c. Solve each problem.

1. Michelle and Corazon are comparing their heights. If Michelle’s height is 120 \( \frac{3}{4} \) cm. and Corazon’s height is 96 \( \frac{1}{3} \) cm. What is the difference in their heights?

   Answer: 24 \( \frac{5}{12} \) cm

2. Angel bought 6 \( \frac{3}{4} \) meters of silk, 3 \( \frac{1}{2} \) meters of satin and 8 \( \frac{2}{5} \) meters of velvet. How many meters of cloth did she buy? Answer: 18 \( \frac{13}{20} \) m

3. Arah needs 10 \( \frac{1}{4} \) kg. of meat to serve 55 guests. If she has 3 \( \frac{1}{2} \) kg of chicken, a 2 \( \frac{3}{4} \) kg of pork, and 4 \( \frac{1}{4} \) kg of beef, is there enough meat for 55 guests? Answer: Yes, she has enough. She has a total of 10 \( \frac{1}{2} \) kilos.

4. Mr. Tan has 13 \( \frac{2}{5} \) liters of gasoline in his car. He wants to travel far so he added 16 \( \frac{1}{2} \) liters more. How many liters of gasoline is in the tank? Answer: 29 \( \frac{9}{10} \) liters
5. After boiling, the \(17\frac{3}{4}\) liters of water was reduced to \(9\frac{2}{3}\) liters. How much water has evaporated? **Answer:** \(8\frac{1}{12}\) liters

**NOTE TO THE TEACHER:**
The last portion of this module is on the addition and subtraction of rational numbers in decimal form. This is mainly a review but emphasize that they are not just working on decimal numbers but with rational numbers. Emphasize that these decimal numbers are a result of the numerator being divided by the denominator of a quotient of two integers.

### C. Addition and Subtraction of Rational Numbers in Decimal Form

There are 2 ways of adding or subtracting decimals.

1. Express the decimal numbers in fractions then add or subtract as described earlier.

   **Example:**
   
   Add: \(2.3 + 7.21\)  
   \[
   \frac{2\frac{3}{10}}{\frac{9}{10}} + \frac{7\frac{21}{100}}{\frac{721}{100}} = \frac{9\frac{6}{10}}{\frac{96}{100}} - \frac{3\frac{25}{100}}{\frac{325}{100}}
   \]

2. Arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract as with whole numbers.

   **Example:**
   
   Add: \(2.3 + 7.21\)  
   \[
   \frac{2.3}{9.6} + \frac{7.21}{3.25} = \frac{9.51}{6.35}
   \]

### Exercises:

1. Perform the indicated operation.
   1) \(1,902 + 21.36 + 8.7 = 1,932.06\)  
   6) \(700 - 678.891 = 21.109\)
   2) \(45.08 + 9.2 + 30.545 = 84.825\)  
   7) \(7.3 - 5.182 = 2.118\)
2. Solve the following problems:
   a. Helen had P7500 for shopping money. When she got home, she had P132.75 in her pocket. How much did she spend for shopping? **P7367.25**
   b. Ken contributed P69.25, while John and Hanna gave P56.25 each for their gift to Teacher Daisy. How much were they able to gather altogether? **P181.75**
   c. Ryan said, “I’m thinking of a number N. If I subtract 10.34 from N, the difference is 1.34.” What was Ryan’s number? **11.68**
   d. Agnes said, “I’m thinking of a number N. If I increase my number by 56.2, the sum is 14.62.” What was Agnes number? **–41.58**
   e. Kim ran the 100-meter race in 135.46 seconds. Tyron ran faster by 15.7 seconds. What was Tyron’s time for the 100-meter dash? **119.76**

NOTE TO THE TEACHER:
The summary is important especially because this is a long module. This lesson provided students with plenty of exercises to help them master addition and subtraction of rational numbers.

SUMMARY
This lesson began with some activities and instruction on how to change rational numbers from one form to another and proceeded to discuss addition and subtraction of rational numbers. The exercises given were not purely computational. There were thought questions and problem solving activities that helped in deepening one’s understanding of rational numbers.
Lesson 8: Multiplication and Division of Rational Numbers
Time: 2 hours

Prerequisite Concepts: addition and subtraction of rational numbers, expressing rational numbers in different forms

Objectives:
In this lesson, you are expected to:
1. Multiply rational numbers;
2. Divide rational numbers;
3. Solve problems involving multiplication and division of rational numbers.

NOTE TO THE TEACHER:
This lesson reinforces what they learned in elementary mathematics. It starts with the visualization of the multiplication and division of rational numbers using the area model. Use different, yet appropriate shapes when illustrating using the area model. The opening activity encourages the students to use a model or drawing to help them solve the problem. Although, some students will insist they know the answer, it is a whole different skill to teach them to visualize using the area model.

Lesson Proper
A. Models for the Multiplication and Division
   I. Activity:
      Make a model or a drawing to show the following:
      1. A pizza is divided into 10 equal slices. Kim ate \( \frac{3}{5} \) of \( \frac{1}{2} \) of the pizza. What part of the whole pizza did Kim eat?
      2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

      Can you make a model or a drawing to help you solve these problems?

      A model that we can use to illustrate multiplication and division of rational numbers is the area model.

      What is \( \frac{1}{4} \times \frac{1}{3} \)? Suppose we have one bar of chocolate represent 1 unit.

      Divide the bar first into 4 equal parts vertically. One part of it is \( \frac{1}{4} \).
Then, divide each fourth into 3 equal parts, this time horizontally to make the divisions easy to see. One part of the horizontal division is $\frac{1}{3}$.

There will be 12 equal-sized pieces and one piece is $\frac{1}{12}$. But, that one piece is $\frac{1}{3}$ of $\frac{1}{4}$, which we know from elementary mathematics to mean $\frac{1}{3} \times \frac{1}{4}$.

**NOTE TO THE TEACHER**

The area model is also used in visualizing division of rational numbers in fraction form. This can be helpful for some students. For others, the model may not be easily understandable. But, do not give up. It is a matter of getting used to. In fact, this is a good way to help them use a non-algorithmic approach to dividing rational numbers in fraction form: by using the idea that division is the reverse of multiplication.

What about a model for division of rational numbers?

Take the division problem: $\frac{4}{5} \div \frac{1}{2}$. One unit is divided into 5 equal parts and 4 of them are shaded.

Each of the 4 parts now will be cut up in halves

Since there are 2 divisions per part (i.e. $\frac{1}{2}$) and there are 4 of them (i.e. $\frac{4}{5}$), then there will be 8 pieces out of 5 original pieces or $\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$. 
NOTE TO THE TEACHER

The solution to the problem $\frac{4}{5} \div \frac{1}{2}$ can be easily checked using the area model as well. Ask the students, what is $\frac{1}{2} \times \frac{8}{5}$. The answer can be obtained using the area model.

$\frac{1}{2} \times \frac{8}{5} = \frac{4}{5}$

NOTE TO THE TEACHER:

It is important for you to go over the answers of your students to the questions posed in the opening activity in order to process what they have learned for themselves. Encourage discussions and exchanges in the class. Do not leave questions unanswered.

II. Questions to Ponder (Post-Activity Discussion)

Let us answer the questions posed in the opening activity.

1. A pizza is divided into 10 equal slices. Kim ate $\frac{3}{5}$ of $\frac{1}{2}$ of the pizza. What part of the whole pizza did Kim eat?

$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

$\frac{1}{2}$

$\frac{3}{10}$

Kim ate $\frac{3}{10}$ of the whole pizza.

NOTE TO THE TEACHER

The area model works for multiplication of rational numbers because the operation is binary, meaning it is an operation done on two elements. The area model allows for at most “shading” or “slicing” in two directions.
2. Miriam made 8 chicken sandwiches for some street children. She cut up each sandwich into 4 triangular pieces. If a child can only take a piece, how many children can she feed?

The equation is \( 8 \div \frac{1}{4} = 32 \). Since there are 4 fourths in one sandwich, there will be \( 4 \times 8 = 32 \) triangular pieces and hence, 32 children will be fed.

How then can you multiply or divide rational numbers without using models or drawings?

**NOTE TO THE TEACHER:**
Below are important rules or procedures that the students must remember. From here on, be consistent in your rules so that your students will not be confused. Give plenty of examples.

*Important Rules to Remember*

The following are rules that you must remember. From here on, the symbols to be used for multiplication are any of the following: \( \bullet, \times, \times, \text{ or } \times \).

1. To multiply rational numbers in fraction form simply multiply the numerators and multiply the denominators.

   \[
   \frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}
   \]

   where: \( b \) and \( d \) are NOT equal to zero, \( (b \neq 0; d \neq 0) \)

2. To divide rational numbers in fraction form, you take the reciprocal of the second fraction (called the divisor) and multiply it by the first fraction.
In symbol, \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \) where: b, c, and d are NOT equal to zero.

Example:
Multiply the following and write your answer in simplest form

a. \( \frac{3}{7} \cdot \frac{2}{5} \)
\[ \frac{3}{7} \cdot \frac{2}{5} = \frac{3 \times 2}{7 \times 5} = \frac{6}{35} \]

b. \( 4\frac{1}{3} \cdot 2\frac{1}{4} \)
\[ 4\frac{1}{3} \cdot 2\frac{1}{4} = \frac{13}{3} \cdot \frac{9}{4} = \frac{13 \cdot 3}{3 \cdot 4} = \frac{39}{4} \text{ or } 9\frac{3}{4} \]

The easiest way to solve for this number is to change mixed numbers to an improper fraction and then multiply it. Or use prime factors or the greatest common factor, as part of the multiplication process.

Divide:
\[ \frac{8}{11} \div \frac{2}{3} = \frac{8}{11} \cdot \frac{3}{2} = \frac{2 \cdot 4 \cdot 3}{11} = \frac{12}{11} \text{ or } 1\frac{1}{11} \]

Take the reciprocal of \( \frac{2}{3} \), which is \( \frac{3}{2} \) then multiply it with the first fraction. Using prime factors, it is easy to see that 2 can be factored out of the numerator then cancelled out with the denominator, leaving 4 and 3 as the remaining factors in the numerator and 11 as the remaining factors in the denominator.

III. Exercises.
Do the following exercises. Write your answer on the spaces provided:
1. Find the products. Express in lowest terms (i.e. the numerator and denominators do not have a common factor except 1). Mixed numbers are acceptable as well:

a. \( \frac{5}{6} \cdot \frac{2}{3} = \frac{5}{9} \)

b. \( 7 \cdot \frac{2}{3} = \frac{14}{3} = 4\frac{2}{3} \)

c. \( \frac{4}{20} \cdot \frac{2}{5} = \frac{2}{25} \)

d. \( 10\frac{5}{6} \cdot 3\frac{1}{3} = \frac{325}{9} = 36\frac{1}{9} \)

e. \( -\frac{9}{20} \cdot \frac{25}{27} = -\frac{5}{12} \)

f. \( 4\frac{1}{2} \cdot 5\frac{2}{3} = \frac{51}{2} = 25\frac{1}{2} \)

g. \( \frac{2}{15} \cdot \frac{3}{4} = \frac{1}{10} \)

h. \( \frac{1}{6} \cdot \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{36} \)

i. \( -\frac{5}{6} \cdot \frac{2}{3} \cdot \left(-\frac{12}{15}\right) = \frac{4}{9} \)

j. \( \frac{9}{16} \cdot \frac{4}{15} \cdot (-2) = -\frac{3}{10} \)
B. Divide:

1. \( 20 \div \frac{2}{3} = 30 \)
2. \( \frac{5}{12} + \left( -\frac{3}{4} \right) = -\frac{5}{9} \)
3. \( \frac{5}{50} \div \frac{20}{35} = \frac{7}{40} \)
4. \( \frac{5}{4} \div \frac{6}{3} = \frac{69}{80} \)
5. \( \frac{9}{16} \div \frac{3}{4} \div \frac{1}{6} = 2 \frac{1}{2} \)
6. \( \frac{8}{15} \div \frac{12}{25} = \frac{10}{9} = 1\frac{1}{9} \)
7. \( 13\frac{1}{6} \div (-2) = -\frac{79}{12} = -6\frac{7}{12} \)
8. \( -\frac{5}{6} \div \left( -\frac{10}{14} \right) = \frac{7}{6} = 1\frac{1}{6} \)
9. \( -\frac{2}{9} \div \frac{11}{15} = -\frac{10}{33} \)
10. \( \frac{15}{6} \div \frac{2}{3} \div \frac{5}{8} = 6 \)

C. Solve the following:

1. Julie spent \( 3\frac{1}{2} \) hours doing her assignment. Ken did his assignment for \( 1\frac{2}{3} \) times as many hours as Julie did. How many hours did Ken spend doing his assignment? \( \frac{35}{6} = 5\frac{5}{6} \) hours
2. How many thirds are there in six-fifths? \( \frac{18}{5} = 3\frac{3}{5} \)
3. Hanna donated \( \frac{2}{5} \) of her monthly allowance to the Iligan survivors. If her monthly allowance is \( P3500 \), how much did she donate? \( P1,400.00 \)
4. The enrolment for this school year is 2340. If \( \frac{1}{6} \) are sophomores and \( \frac{1}{4} \) are seniors, how many are freshmen or juniors? 1,365 students are freshmen or juniors
5. At the end of the day, a store had \( \frac{2}{5} \) of a cake leftover. The four employees each took home the same amount of leftover cake. How much of the cake did each employee take home? \( \frac{1}{10} \) of the cake.

B. Multiplication and Division of Rational Numbers in Decimal Form

NOTE TO THE TEACHER

The emphasis here is on what to do with the decimal point when multiplying or dividing rational numbers in decimal form. Do not get stuck on the rules. Give a deeper explanation. Consider:

\[ 6.1 \times 0.08 = 6 \frac{1}{10} \times \frac{8}{100} = \frac{488}{1000} = 0.488 \]
The decimal places indicate the powers of 10 used in the denominators hence, the rule for determining where to place the decimal point in the product.

This unit will draw upon your previous knowledge of multiplication and division of whole numbers. Recall the strategies that you learned and developed when working with whole numbers.

Activity:
1. Give students several examples of multiplication sentences with the answers given. Place the decimal point in an incorrect spot and ask students to explain why the decimal place does not go there and explain where it should go and why.
   Example:
   
   $$215.2 \times 3.2 = 68.864$$

2. Five students ordered buko pie and the total cost was P135.75. How much did each student have to pay if they shared the cost equally?

Questions and Points to Ponder:
1. In multiplying rational numbers in decimal form, note the importance of knowing where to place the decimal point in a product of two decimal numbers. Do you notice a pattern? Take the sum of the decimal places in each of the multiplicand and the multiplier and that is the number of places in the product.
2. In dividing rational numbers in decimal form, how do you determine where to place the decimal point in the quotient? The number of decimal places in the quotient depends on the number of decimal places in the divisor and the dividend.

NOTE TO THE TEACHER
Answer to the Questions and Points to Ponder is to be elaborated when you discuss the rules below.

Rules in Multiplying Rational Numbers in Decimal Form
1. Arrange the numbers in a vertical column.
2. Multiply the numbers, as if you are multiplying whole numbers.
3. Starting from the rightmost end of the product, move the decimal point to the left the same number of places as the sum of the decimal places in the multiplicand and the multiplier.

Rules in Dividing Rational Numbers in Decimal Form
1. If the divisor is a whole number, divide the dividend by the divisor applying the rules of a whole number. The position of the decimal point is the same as that in the dividend.
2. If the divisor is not a whole number, make the divisor a whole number by moving the decimal point in the divisor to the rightmost end, making the number seem like a whole number.
3. Move the decimal point in the dividend to the right the same number of places as the decimal point was moved to make the divisor a whole number.
4. Lastly divide the new dividend by the new divisor.

**Exercises:**
A. Perform the indicated operation

1. 3.5 ÷ 2 = 1.75
2. 78 ÷ 0.4 = 195
3. 9.6 × 13 = 124.8
4. 3.24 ÷ 0.5 = 6.48
5. 1.248 ÷ 0.024 = 52
6. 27.3 × 2.5 = 68.25
7. 9.7 × 4.1 = 39.77
8. 3.415 ÷ 2.5 = 1.366
9. 27.3 × 2.5 = 68.25
10. 9.7 × 4.1 = 39.77

B. Finds the numbers that when multiplied give the products shown.

1. \[ x \quad \] \[ \times \quad 1 \quad 0 \quad . \quad 6 \]
2. \[ x \quad \] \[ \times \quad 1 \quad 6 \quad . \quad 8 \]
3. \[ x \quad \] \[ \times \quad 2 \quad 1 \quad . \quad 6 \]
4. \[ x \quad \] \[ \times \quad 2 \quad 1 \quad . \quad 9 \quad . \quad 8 \]

Answers: (1) 5.3 \times 2; (2) 8.4 \times 2 or 5.6 \times 3; (3) 5.4 \times 4; (4) 3.5 \times 3; (5) 3.14 \times 7

NOTE TO THE TEACHER: These are only some of the possible pairs. Be open to other pairs of numbers.

**NOTE TO THE TEACHER**

Give a good summary to this lesson emphasizing how this lesson was meant to deepen their understanding of rational numbers and develop better skills in multiplying and dividing rational numbers.

**Summary**

In this lesson, you learned to use the area model to illustrate multiplication and division of rational numbers. You also learned the rules for multiplying and dividing rational numbers in both the fraction and decimal forms. You solved problems involving multiplication and division of rational numbers.
Lesson 9: Properties of the Operations on Rational Numbers  
Time: 1 hour

Pre-requisite Concepts: Operations on rational numbers

Objectives:
In this lesson, you are expected to
1. Describe and illustrate the different properties of the operations on rational numbers.
2. Apply the properties in performing operations on rational numbers.

NOTE TO THE TEACHER:
Generally, rational numbers appear difficult among students. The following activity should be fun and could help your students realize the importance of the properties of operations on rational numbers.

Lesson Proper:
I. Activity

Pick a Pair

<table>
<thead>
<tr>
<th>2</th>
<th>3</th>
<th>0</th>
<th>1</th>
<th>13</th>
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<tbody>
<tr>
<td>14</td>
<td>5</td>
<td></td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>3</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the box above, pick the correct rational number to be placed in the spaces provided to make the equation true.

1. \( \frac{3}{14} + \frac{2}{14} = \frac{5}{14} \)
2. \( \frac{2}{14} + \frac{3}{14} = \frac{5}{14} \)
3. \( \frac{1}{3} \times \_ = 0 \) [0]
4. \( 1 \times \frac{3}{5} = \frac{3}{5} \)
5. \( \frac{2}{3} + [0] = \frac{2}{3} \)
6. \( \frac{1}{2} + \frac{1}{4} + \frac{1}{3} = \frac{13}{12} \)
7. \( \frac{1}{2} + \frac{1}{4} + \_ = \frac{13}{12} \)
8. \( \frac{2}{5} \times \left( \_ \times \frac{3}{4} \right) = \frac{3}{20} \) [\( \frac{1}{5} \)]
9. \( \left( \frac{2}{5} \times \frac{1}{2} \right) \times \frac{3}{4} = \_ \) [\( \frac{3}{20} \)]
10. \( \frac{1}{2} \times \left( \frac{2}{5} + \frac{1}{4} \right) = \left( \frac{1}{2} \times \frac{2}{5} \right) + \left( \frac{1}{2} \times \frac{1}{4} \right) = \frac{13}{40} \)

Answer the following questions:
1. What is the missing number in item 1?
2. How do you compare the answers in items 1 and 2?
3. What about item 3? What is the missing number?
4. In item 4, what number did you multiply with 1 to get \( \frac{3}{5} \)?
5. What number should be added to $\frac{2}{3}$ in item 5 to get the same number?
6. What is the missing number in items 6 and 7?
7. What can you say about the grouping in items 6 and 7?
8. What do you think are the answers in items 8 and 9?
9. What operation did you apply in item 10?

NOTE TO THE TEACHER
The follow-up problem below could make the points raised in the previous activity clearer.

**Problem:**
Consider the given expressions:

a. \[ \frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{2}{3} = \frac{1}{4} + \frac{1}{2} + \frac{2}{3} + \frac{1}{8} \]

b. \[ \frac{2}{15} \cdot \frac{5}{6} = \frac{5}{6} \cdot \frac{2}{15} \]

* Are the two expressions equal? If yes, state the property illustrated. Yes, the expressions in item (a) are equal and so are the expressions in item (b). This is due to the Commutative Property of Addition and of Multiplication. The Commutative Property allows you to change the order of the addends or factors and the resulting sum or product, respectively, will not change.

NOTE TO THE TEACHER
Discuss among your students the following properties. These properties make adding and multiplying of rational numbers easier to do.

**PROPERTIES OF RATIONAL NUMBERS (ADDITION & MULTIPLICATION)**

1. **CLOSURE PROPERTY:** For any two defined rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, their sum $\frac{a}{b} + \frac{c}{d}$ and product $\frac{a}{b} \cdot \frac{c}{d}$ is also rational.

   For example:
   a. \[ \frac{3}{4} + \frac{2}{4} = \frac{3+2}{4} = \frac{5}{4} \]
   b. \[ \frac{3}{4} \cdot \frac{2}{4} = \frac{6}{16} \text{ or } \frac{3}{8} \]

2. **COMMUTATIVE PROPERTY:** For any two defined rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

   i. \[ \frac{a}{b} + \frac{c}{d} = \frac{b}{d} + \frac{a}{b} \]
   ii. \[ \frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \text{ and } \frac{a}{b} \]
For example:

a. \[ \frac{2}{7} + \frac{1}{3} = \frac{1}{3} + \frac{1}{7} \]

b. \[ \frac{6}{7} \cdot \frac{2}{3} = \frac{3}{3} \cdot \frac{6}{7} \]

3. ASSOCIATIVE PROPERTY: For any three defined rational numbers \( \frac{a}{b}, \frac{c}{d}, \) and \( \frac{e}{f} \)

   i. \[ \frac{a}{b} + \left( \frac{c}{d} + \frac{e}{f} \right) = \left( \frac{a}{b} + \frac{c}{d} \right) + \frac{e}{f} \]

   ii. \[ \frac{a}{b} \cdot \left( \frac{c}{d} \cdot \frac{e}{f} \right) = \left( \frac{a}{b} \cdot \frac{c}{d} \right) \cdot \frac{e}{f} \]

For example:

a. \[ \frac{3}{5} + \left( \frac{2}{3} + \frac{1}{4} \right) = \left( \frac{3}{5} + \frac{2}{3} \right) + \frac{1}{4} \]

b. \[ \frac{1}{4} \cdot \left( \frac{3}{4} \cdot \frac{2}{3} \right) = \left( \frac{1}{4} \cdot \frac{3}{4} \right) \cdot \frac{2}{3} \]

4. DISTRIBUTIVE PROPERTY of multiplication over addition for rational numbers.

If \( \frac{a}{b}, \frac{c}{d}, \text{ and } \frac{e}{f} \) are any defined rational numbers, then \[ \frac{a}{b} \cdot \left( \frac{c}{d} + \frac{e}{f} \right) = \left( \frac{a}{b} \cdot \frac{c}{d} \right) + \left( \frac{a}{b} \cdot \frac{e}{f} \right) \]

For example: \[ \frac{3}{7} \cdot \left( \frac{2}{3} + \frac{7}{8} \right) = \left( \frac{3}{7} \cdot \frac{2}{3} \right) + \left( \frac{3}{7} \cdot \frac{7}{8} \right) \]

5. DISTRIBUTIVE PROPERTY of multiplication over subtraction for rational numbers.

If \( \frac{a}{b}, \frac{c}{d}, \text{ and } \frac{e}{f} \) are any defined rational numbers, then \[ \frac{a}{b} \cdot \left( \frac{c}{d} - \frac{e}{f} \right) = \left( \frac{a}{b} \cdot \frac{c}{d} \right) - \left( \frac{a}{b} \cdot \frac{e}{f} \right) \]

For example: \[ \frac{3}{10} \cdot \left( \frac{2}{3} - \frac{2}{8} \right) = \left( \frac{3}{10} \cdot \frac{2}{3} \right) - \left( \frac{3}{10} \cdot \frac{2}{8} \right) \]

6. IDENTITITY PROPERTY

Addition: Adding 0 to a number will not change the identity or value of that number.

\[ \frac{a}{b} + 0 = \frac{a}{b} \]

For example: \[ \frac{1}{2} + 0 = \frac{1}{2} \]
Multiplication: Multiplying a number by 1 will not change the identity or value of that number.
\[
\frac{a}{b} \cdot 1 = \frac{a}{b}
\]

For example: \( \frac{3}{5} \cdot 1 = \frac{3}{5} \)

7. ZERO PROPERTY OF MULTIPLICATION: Any number multiplied by zero equals 0, i.e.
\[
\frac{a}{b} \cdot 0 = 0
\]

For example: \( \frac{2}{7} \cdot 0 = 0 \)

II. Question to Ponder (Post-Activity Discussion)

**NOTE TO THE TEACHER**
Answer each question in the opening Activity thoroughly and discussed the concepts clearly. Allow students to express their ideas, their doubts and their questions. At this stage, they should really be able to verbalize what they understand or do not understand so that you the teacher may properly address any misconceptions they have. Give plenty of additional examples, if necessary.

Let us answer the questions posed in the opening activity.

1. What is the missing number in item 1? » \( \frac{2}{14} \)
2. How do you compare the answers in items 1 and 2? » The answer is the same, the order of the numbers is not important.
3. What about item 3? What is the missing number? » The missing number is 0. When you multiply a number with zero the product is zero.
4. In item 4, what number did you multiply with 1 to get \( \frac{3}{5} \)? » \( \frac{3}{5} \), When you multiply a number by one the answer is the same.
5. What number should be added to \( \frac{2}{3} \) in item 5 to get the same number? » 0, When you add zero to any number, the value of the number does not change.
6. What do you think is the missing number in items 6 and 7? » \( \frac{13}{12} \)
7. What can you say about the grouping in items 6 and 7? » The groupings are different but they do not affect the sum.
8. What do you think are the answers in items 8 and 9? » The answer is the same in both items, \( \frac{3}{20} \).
9. What operation did you apply in item 10? » The Distributive Property of Multiplication over Addition
III. Exercises:

Do the following exercises. Write your answer in the spaces provided.

A. State the property that justifies each of the following statements.

1. \( \frac{2}{3} + \frac{5}{8} = \frac{5}{8} + \frac{2}{3} \)
   Commutative Property

2. \( 1 \times \frac{9}{35} = \frac{9}{35} \)
   Identity Property for Multiplication

3. \( \frac{4}{5} \cdot \left( \frac{3}{4} + \frac{2}{3} \right) = \left( \frac{4}{5} \cdot \frac{3}{4} \right) + \left( \frac{4}{5} \cdot \frac{2}{3} \right) \)
   Distributive Property of Multiplication over Addition

4. \( \frac{3}{5} + \left( \frac{1}{2} + \frac{1}{4} \right) = \left( \frac{3}{5} + \frac{1}{2} \right) + \frac{1}{4} \)
   Associative Property

5. \( \left( \frac{2}{3} + \frac{1}{2} + \frac{3}{4} \right) \cdot 1 = \left( \frac{2}{3} + \frac{1}{2} + \frac{3}{4} \right) \)
   Identity Property for Multiplication

6. \( \left( \frac{3}{4} + 0 \right) = \frac{3}{4} \)
   Identity Property for Addition

7. \( \frac{1}{2} + \frac{5}{6} = \frac{4}{3} \)
   Closure Property

8. \( \frac{3}{8} \cdot \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{3}{8} \cdot \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{2} \)
   Commutative Property

9. \( \frac{1}{4} \cdot \left( \frac{5}{7} - \frac{2}{3} \right) = \left( \frac{1}{4} \cdot \frac{5}{7} \right) - \left( \frac{1}{4} \cdot \frac{2}{3} \right) \)
   Distributive Property of Multiplication over Subtraction

10. \( \left( \frac{2}{15} \cdot \frac{5}{7} \right) \cdot 0 = 0 \)
    Zero Property for Multiplication

B. Find the value of N in each expression

1. \( N + \frac{1}{45} = \frac{1}{45} \quad \Rightarrow \quad N = 0 \)

2. \( \left( \frac{1}{4} \cdot N \right) \cdot \frac{2}{3} = \frac{1}{4} \cdot \left( \frac{6}{7} \cdot \frac{2}{3} \right) \quad \Rightarrow \quad N = \frac{6}{7} \)

3. \( \left( \frac{2}{15} + \frac{12}{30} \right) + \frac{1}{5} = \frac{2}{15} + \left( N + \frac{1}{5} \right) \quad \Rightarrow \quad N = \frac{12}{30} \)
4. \(0 + N = \frac{5}{18}\) \(N = \frac{5}{18}\)

6. \(N \cdot \left(\frac{6}{14} + \frac{2}{7}\right) = \left(\frac{1}{6} \cdot \frac{6}{14}\right) + \left(\frac{1}{6} \cdot \frac{2}{7}\right)\) \(N = \frac{1}{6}\)

7. \(\frac{8}{23} \cdot 1 = N\) \(N = \frac{8}{23}\)

8. \(\frac{2}{9} + \frac{2}{3} = N\) \(N = \frac{8}{9}\)

**NOTE TO THE TEACHER**

You might want to add more exercises. When you are sure that your students have mastered the properties, do not forget to end your lesson with a summary.

**Summary**

This lesson is about the properties of operations on rational numbers. The properties are useful because they simplify computations on rational numbers. These properties are true under the operations addition and multiplication. Note that for the Distributive Property of Multiplication over Subtraction, subtraction is considered part of addition. Think of subtraction as the addition of a negative rational number.
Lesson 10: Principal Roots and Irrational Numbers
Time: 2 hours

Prerequisite Concepts: Set of rational numbers

Objectives:
In this lesson, you are expected to:
1. describe and define irrational numbers;
2. describe principal roots and tell whether they are rational or irrational;
3. determine between what two integers the square root of a number is;
4. estimate the square root of a number to the nearest tenth;
5. illustrate and graph irrational numbers (square roots) on a number line with and without appropriate technology.

NOTE TO THE TEACHER
This is the first time that students will learn about irrational numbers. Irrational numbers are simply numbers that are not rational. However, they are not easy to determine, hence we limit our discussions to principal nth roots, particularly square roots. A lesson on irrational numbers is important because these numbers are often encountered. While the activities are meant to introduce these numbers in a non-threatening way, try not to deviate from the formal discussion on principal nth roots. The definitions are precise so be careful not to overextend or over generalize.

Lesson Proper:
I. Activities
A. Take a look at the unusual wristwatch and answer the questions below.

1. Can you tell the time?
2. What time is shown in the wristwatch?
3. What do you get when you take the √1 ? √4 ? √9 ? √16 ?
4. How will you describe the result?
5. Can you take the exact value of √130 ?
6. What value could you get?

NOTE TO THE TEACHER
In this part of the lesson, the square root of a number is used to introduce a new set of numbers called the irrational numbers. Take note of the two ways by which irrational numbers are described and defined.

Taking the square root of a number is like doing the reverse operation of squaring a number. For example, both 7 and -7 are square roots of 49 since \(7^2 = 49\) and \((-7)^2 = 49\). Integers such as 1, 4, 9, 16, 25 and 36 are called perfect squares. Rational numbers such as 0.16, \(\frac{4}{100}\) and 4.84 are also, perfect squares. Perfect squares are numbers that have rational numbers as square roots. The square roots
of perfect squares are rational numbers while the square roots of numbers that are not perfect squares are irrational numbers.

Any number that cannot be expressed as a quotient of two integers is an irrational number. The numbers $\sqrt{2}$, $\pi$, and the special number $e$ are all irrational numbers. Decimal numbers that are non-repeating and non-terminating are irrational numbers.

NOTE TO THE TEACHER
It does not hurt for students at this level to use a scientific calculator in obtaining principal roots of numbers. With the calculator, it becomes easier to identify as well irrational numbers.

B. Activity
Use the $\sqrt[n]{ }$ button of a scientific calculator to find the following values:
1. $\sqrt[6]{64}$
2. $\sqrt[4]{-16}$
3. $\sqrt[90]{90}$
4. $\sqrt[3125]{-3125}$
5. $\sqrt[24]{24}$

II. Questions to Ponder (Post-Activity Discussions)
Let us answer the questions in the opening activity.
1. Can you tell the time? Yes
2. What time is it in the wristwatch? 10:07
3. What do you get when you take the $\sqrt{1}$? $\sqrt{4}$? $\sqrt{9}$? $\sqrt{16}$? 1, 2, 3, 4
4. How will you describe the result? They are all positive integers.
5. Can you take the exact value of $\sqrt{130}$? No.
6. What value could you get? Since the number is not a perfect square you could estimate the value to be between $\sqrt{121}$ and $\sqrt{144}$, which is about 11.4.

Let us give the values asked for in Activity B. Using a scientific calculator, you probably obtained the following:
1. $\sqrt[6]{64} = 2$
2. $\sqrt[4]{-16}$ Math Error, which means not defined
3. $\sqrt[90]{90} = 4.481404747$, which could mean non-terminating and non-repeating since the calculator screen has a limited size
4. $\sqrt[3125]{-3125} = -5$
5. $\sqrt[24]{24} = 4.898979486$, which could mean non-terminating and non-repeating since the calculator screen has a limited size

NOTE TO THE TEACHER
The transition from the concept of two square roots of a positive number to that of the principal $nth$ root has always been a difficult one for students. The important and precisely stated concepts are in bold so that students pay attention to them. Solved problems that are meant to illustrate certain procedures and techniques in determining whether a principal root is rational or irrational, finding two consecutive integers between which the
An irrational number is found, estimating the value of irrational square roots to the nearest tenth, and plotting an irrational square root on a number line.

**On Principal n<sup>th</sup> Roots**

Any number, say \( a \), whose \( n \)-th power (\( n \), a positive integer), is called the \( n \)-th root of \( b \). Consider the following: \((-7)^2 = 49\), \( 2^4 = 16 \) and \((-10)^3 = -1000\). This means that -7 is a 2<sup>nd</sup> or square root of 49, 2 is a 4<sup>th</sup> root of 16 and -10 is a 3<sup>rd</sup> or cube root of -1000.

However, we are not simply interested in any \( n \)-th root of a number; we are more concerned about the principal \( n \)-th root of a number. The principal \( n \)-th root of a positive number is the positive \( n \)-th root. The principal \( n \)-th root of a negative number is the negative \( n \)-th root if \( n \) is odd. If \( n \) is even and the number is negative, the principal \( n \)-th root is not defined. The notation for the principal \( n \)-th root of a number \( b \) is \( \sqrt[n]{b} \). In this expression, \( n \) is the index and \( b \) is the radicand. The \( n \)-th roots are also called radicals.

**Classifying Principal n<sup>th</sup> Roots as Rational or Irrational Numbers**

To determine whether a principal root is a rational or irrational number, determine if the radicand is a perfect \( n \)-th power of a number. If it is, then the root is rational. Otherwise, it is irrational.

**Problem 1.** Tell whether the principal root of each number is rational or irrational.

(a) \( \sqrt[3]{225} \)  
(b) \( \sqrt{0.04} \)  
(c) \( \sqrt[5]{-111} \)  
(d) \( \sqrt{10000} \)  
(e) \( \sqrt[4]{625} \)

**Answers:**

a) \( \sqrt[3]{225} \) is irrational  
(b) \( \sqrt{0.04} = 0.2 \) is rational  
(c) \( \sqrt[5]{-111} \) is irrational  
(d) \( \sqrt{10000} = 100 \) is rational  
(e) \( \sqrt[4]{625} = 5 \) is rational

If a principal root is irrational, the best you can do for now is to give an estimate of its value. Estimating is very important for all principal roots that are not roots of perfect \( n \)-th powers.

**Problem 2.** The principal roots below are between two integers. Find the two closest such integers.

(a) \( \sqrt{19} \)  
(b) \( \sqrt{101} \)  
(c) \( \sqrt{300} \)

**Solution:**

(a) \( \sqrt{19} \)

16 is a perfect integer square and 4 is its principal square root. 25 is the next perfect integer square and 5 is its principal square root. Therefore, \( \sqrt{19} \) is between 4 and 5.
(b) \(\sqrt[3]{101}\)

64 is a perfect integer cube and 4 is its principal cube root. 125 is the next perfect integer cube and 5 is its principal cube root. Therefore, \(\sqrt[3]{101}\) is between 4 and 5.

(c) \(\sqrt{300}\)

289 is a perfect integer square and 17 is its principal square root. 324 is the next perfect integer square and 18 is its principal square root. Therefore, \(\sqrt{300}\) is between 17 and 18.

**Problem 3.** Estimate each square root to the nearest tenth.

(a) \(\sqrt{40}\)  
(b) \(\sqrt{12}\)  
(c) \(\sqrt{175}\)

**Solution:**

(a) \(\sqrt{40}\)

The principal root \(\sqrt{40}\) is between 6 and 7, principal roots of the two perfect squares 36 and 49, respectively. Now, take the square of 6.5, midway between 6 and 7. Computing, \((6.5)^2 = 42.25\). Since 42.25 > 40 then \(\sqrt{40}\) is closer to 6 than to 7. Now, compute for the squares of numbers between 6 and 6.5: \((6.1)^2 = 37.21\), \((6.2)^2 = 38.44\), \((6.3)^2 = 39.69\), and \((6.4)^2 = 40.96\). Since 40 is close to 39.69 than to 40.96, \(\sqrt{40}\) is approximately 6.3.

(b) \(\sqrt{12}\)

The principal root \(\sqrt{12}\) is between 3 and 4, principal roots of the two perfect squares 9 and 16, respectively. Now take the square of 3.5, midway between 3 and 4. Computing \((3.5)^2 = 12.25\). Since 12.25 > 12 then \(\sqrt{12}\) is closer to 3 than to 4. Compute for the squares of numbers between 3 and 3.5: \((3.1)^2 = 9.61\), \((3.2)^2 = 10.24\), \((3.3)^2 = 10.89\), and \((3.4)^2 = 11.56\). Since 12 is closer to 12.25 than to 11.56, \(\sqrt{12}\) is approximately 3.5.

(c) \(\sqrt{175}\)

The principal root \(\sqrt{175}\) is between 13 and 14, principal roots of the two perfect squares 169 and 196. The square of 13.5 is 182.25, which is greater than 175. Therefore, \(\sqrt{175}\) is closer to 13 than to 14. Now: \((13.1)^2 = 171.61\), \((13.2)^2 = 174.24\), \((13.3)^2 = 176.89\). Since 175 is closer to 174.24 than to 176.89 then, \(\sqrt{175}\) is approximately 13.2.

**Problem 4.** Locate and plot each square root on a number line.

(a) \(\sqrt{3}\)  
(b) \(\sqrt{21}\)  
(c) \(\sqrt{87}\)

**Solution:** You may use a program like Geogebra to plot the square roots on a number line.
(a) $\sqrt{3}$
This number is between 1 and 2, principal roots of 1 and 4. Since 3 is closer to 4 than to 1, $\sqrt{3}$ is closer to 2. Plot $\sqrt{3}$ closer to 2.

(b) $\sqrt{21}$
This number is between 4 and 5, principal roots of 16 and 25. Since 21 is closer to 25 than to 16, $\sqrt{21}$ is closer to 5 than to 4. Plot $\sqrt{21}$ closer to 5.

(c) $\sqrt{87}$
This number is between 9 and 10, principal roots of 81 and 100. Since 87 is closer to 81, then $\sqrt{87}$ is closer to 9 than to 10. Plot $\sqrt{87}$ closer to 9.

III. Exercises
A. Tell whether the principal roots of each number is rational or irrational.

1. $\sqrt{400}$
2. $\sqrt{64}$
3. $\sqrt{0.01}$
4. $\sqrt{26}$
5. $\frac{1}{\sqrt{49}}$
6. $\sqrt{13.689}$
7. $\sqrt{1000}$
8. $\sqrt{2.25}$
9. $\sqrt{39}$
10. $\sqrt{12.1}$

Answers:
1. rational
2. rational
3. rational
4. irrational
5. rational
6. rational
7. irrational
8. rational
9. irrational
10. irrational

B. Between which two consecutive integers does the square root lie?

1. $\sqrt{77}$
2. $\sqrt{700}$
3. $\sqrt{243}$
4. $\sqrt{444}$
5. $\sqrt{48}$
6. $\sqrt{90}$
7. $\sqrt{2045}$
8. $\sqrt{903}$
9. $\sqrt{1899}$
10. $\sqrt{100000}$
Answers:
1. 8 and 9
2. 26 and 27
3. 15 and 16
4. 21 and 22
5. 6 and 7
6. 9 and 10
7. 45 and 46
8. 30 and 31
9. 43 and 44
10. 316 and 317

C. Estimate each square root to the nearest tenth and plot on a number line.

1. \(\sqrt{6}\)
2. \(\sqrt{7}\)
3. \(\sqrt{8}\)
4. \(\sqrt{9}\)
5. \(\sqrt{10}\)

Answers:
1. 7.1
2. 8.5
3. 3.9
4. 7.3
5. 11.7
6. 15.8
7. 2.2
8. 9.2
9. 6.2
10. 10.0

NOTE TO THE TEACHER

You might think that plotting the irrational square roots on a number line is easy. Do not assume that all students understand what to do. Give them additional exercises for practice. Exercise D can be varied to include 2 or 3 irrational numbers plotted and then asking students to identify the correct graph for the 2 or 3 numbers.

D. Which point on the number line below corresponds to which square root?

\[ \sqrt{57}, \sqrt{6}, \sqrt{99}, \sqrt{38}, \sqrt{10} \]

1. \(\sqrt{57}\) D
2. \(\sqrt{6}\) A
3. \(\sqrt{99}\) E
4. \(\sqrt{38}\) C
5. \(\sqrt{10}\) B

Summary

In this lesson, you learned about irrational numbers and principal \(n^{th}\) roots, particularly square roots of numbers. You learned to find two consecutive integers between which an irrational square root lies. You also learned how to estimate the square roots of numbers to the nearest tenth and how to plot the estimated square roots on a number line.
Lesson 11: The Absolute Value of a Number

Time: 1.5 hours

Prerequisite Concepts: Set of real numbers

Objectives:
In this lesson, you are expected to describe and illustrate
a. the absolute value of a number on a number line.
b. the distance of the number from 0.

Lesson Proper:
I. Activity 1: THE METRO MANILA RAIL TRANSIT (MRT) TOUR
Suppose the MRT stations from Pasay City to Quezon City were on a straight line and were 500 meters apart from each other.
1. How far would the North Avenue station be from Taft Avenue? **6000 meters or 6 kilometers**

2. What if Elaine took the MRT from North Avenue and got off at the last station? How far would she have travelled? **6000 meters or 6 kilometers**

3. Suppose both Archie and Angelica rode the MRT at Shaw Boulevard and the former got off in Ayala while the latter in Kamuning. How far would each have travelled from the starting point to their destinations? **Archie travelled 2000 meters from Shaw Boulevard to Ayala. Angelica travelled 2000 meters from Shaw Boulevard to Kamuning.**

4. What can you say about the directions and the distances travelled by Archie and Angelica? **They went in opposite direction from the same starting point travelled the same distance.**

**NOTE TO THE TEACHER:**

This lesson focuses on the relationship between absolute value and distance. Point out to students that the absolute value of a number as a measure of distance will always be positive or zero since it is simply a magnitude, a measure. Students should realize the importance of the absolute value of a number in contexts such as transportation, weather, statistics and others.

**Activity 2: THE BICYCLE JOY RIDE OF ARCHIE AND ANGELICA**

**Problem:** Archie and Angelica were at Aloys’ house. Angelica rode her bicycle 3 miles west of Aloys’ house, and Archie rode his bicycle 3 miles east of Aloys’ house. Who travelled a greater distance from Aloys’ house – Archie or Angelica?  

**Questions To Ponder:**

1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.
2. What are opposite numbers on the number line? Give examples and show on the number line.

3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?

4. What can you say about the absolute value of opposite numbers say -5 and +5?

5. How can we represent the absolute value of a number? What notation can we use?

**NOTE TO THE TEACHER:**

Below are important terminologies, notations and symbols that your students must learn and remember. From here on, be consistent in using these notations so as not to create confusion on the part of the students. Take note of the subtle difference in using the absolute value bars from the parentheses.

**Important Terms to Remember**

The following are terms that you must remember from this point on.

1. **Absolute Value** – of a number is the distance between that number and zero on the number line.

2. **Number Line** – is best described as a straight line which is extended in both directions as illustrated by arrowheads. A number line consists of three elements:
   a. set of positive numbers, and is located to the right of zero.
   b. set of negative numbers, and is located to the left of zero; and
   c. Zero.

**Notations and Symbols**

The absolute value of a number is denoted by two bars ||.

Let's look at the number line:

```
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 +7 +8 +9 +10
```

The absolute value of a number, denoted "| |", is the distance of the number from zero. This is why the absolute value of a number is never negative. In thinking about the absolute value of a number, one only asks "how far?" not "in which direction?" Therefore, the absolute value of 3 and of -3 is the same, which is 3 because both numbers have the same distance from zero.
Warning: The absolute-value notation is bars, not parentheses or brackets. Use the proper notation; the other notations do not mean the same thing.

It is important to note that the absolute value bars do NOT work in the same way as do parentheses. Whereas \(- (-3) = +3\), this is NOT how it works for absolute value:

**Problem:** Simplify \(- | -3 | \).

**Solution:** Given \(- | -3 | \), first find the absolute value of \(-3\).

\[- | -3 | = - (3)\]

Now take the negative of 3. Thus, :

\[- | -3 | = - (3) = -3\]

This illustrates that if you take the negative of the absolute value of a number, you will get a negative number for your answer.

II. Questions to Ponder (Post-Activity Discussion)

**NOTE TO THE TEACHER**

It is important for you to examine and discuss the responses by your students to the questions posed in Activity 2. Pay particular attention to how they say and write. Always refer to practical examples so they can understand more. Encourage brainstorming, dialogues and arguments in the class. After the exchanges, see to it that all questions are answered and resolved.

Let us answer the questions posed in Activity 2.

1. What subsets of real numbers are used in the problem? Represent the trip of Archie and Angelica to the house of Aloys using a number line.

   The problem uses integers. Travelling 3 miles west can be represented by \(-3\) (pronounced negative 3). Travelling 3 miles east can be represented by \(+3\) (pronounced positive 3). Aloys’ house can be represented by the integer 0.

2. What are opposite numbers on the number line? Give examples and show on the number line.
Two integers that are the same distance from zero in opposite directions are called **opposites**. The integers +3 and -3 are opposites since they are each 3 units from zero.

![Opposites Diagram]

3. What does it mean for the same distance travelled but in opposite directions? How would you interpret using the numbers -3 and +3?

The absolute value of a number is its distance from zero on the number line. The absolute value of +3 is 3, and the absolute value of -3 is 3.

4. What can you say about the absolute value of opposite numbers say -5 and +5?

*Opposite numbers have the same absolute values.*

5. How can we represent the absolute value of a number? What notation can we use?

*The symbol | | is used for the absolute value of a number.*

### III. Exercises

Carry out the following tasks. Write your answers on the spaces provided for each number.

1. Find the absolute value of +3, -3, +7, -5, +9, -8, +4, -4. You may refer to the number line below. What should you remember when we talk about the absolute value of a number?

![Number Line]

Solution: | +3 | = 3  | +9 | = 9  
|---|---|---|---|
| -3 | = 3  | -8 | = 8  
| +7 | = 7  | +4 | = 4  
| -5 | = 5  | -4 | = 4  

Remember that when we find the absolute value of a number, we are
finding its distance from 0 on the number line. Opposite numbers have the same absolute value since they both have the same distance from 0. Also, you will notice that taking the absolute value of a number automatically means taking the positive value of that number.

2. Find the absolute value of: +11, −9, +14, −10, +17, −19, +20, −20.
   You may extend the number line below to help you solve this problem.

Solution:

\[ |+11| = 11 \quad |+17| = 17 \]
\[ |-9| = 9 \quad |-19| = 19 \]
\[ |+14| = 14 \quad |+20| = 20 \]
\[ |-10| = 10 \quad |-20| = 20 \]

3. Use the number line below to find the value of N: \( |N| = 5.1 \)

Solution: This problem asks us to find all numbers that are a distance of 5.1 units from zero on the number line. We let N represent all integers that satisfy this condition.

The number +5.1 is 5.1 units from zero on the number line, and the number −5.1 is also 5.1 units from zero on the number line. Thus both +5.1 and −5.1 satisfy the given condition.

4. When is the absolute value of a number equal to itself?

Solution: When the value of the number is positive or zero.

5. Explain why the absolute value of a number is never negative. Give an example that will support your answer.
Solution: Let $| N | = -4$. Think of a number that when you get the absolute value will give you a negative answer. There will be no solution since the distance of any number from 0 cannot be a negative quantity.

Enrichment Exercises:
A. Simplify the following.
1. $|7.04| = 7.04$
2. $|0| = 0$
3. $|\frac{-5}{9}| = \frac{5}{9}$
4. $-|15 + 6| = -21$
5. $|-2\sqrt{2}| - |-3\sqrt{2}| = -\sqrt{2}$

B. List at least two integers that can replace N such that.
1. $|N| = 4$ \{\(-4, 4\}\}
2. $|N| < 3$ \{-2, -1, 0, 1, 2\}
3. $|N| > 5$ \{\(-\ldots, -10, -9, -8, -7, -6, 6, 7, \ldots\)\}
4. $|N| \leq 9$ \{-9, -8, -7, \ldots, 0, 1, \ldots 9\}
5. $0 < |N| < 3$ \{1, 2\}

C. Answer the following.
1. Insert the correct relation symbol($>$, $=$, $<$): $|-7| > |-4|$.
2. If $|x - 7| = 5$, what are the possible values of $x$? \{2, 12\}
3. If $|x| = \frac{2}{7}$, what are the possible values of $x$? \{\(\frac{2}{7}, \frac{2}{7}\)\}
4. Evaluate the expression, $|x + y| - |y - x|$, if $x = 4$ and $y = 7$. \{8\}
5. A submarine navigates at a depth of 50 meters below sea level while exactly above it; an aircraft flies at an altitude of 185 meters. What is the distance between the two carriers? 235 meters

Summary:
In this lesson you learned about the absolute value of a number, that it is a distance from zero on the number line denoted by the notation $|N|$. This notation is used for the absolute value of an unknown number that satisfies a given condition. You also learned that a distance can never be a negative quantity and absolute value pertains to the magnitude rather than the direction of a number.